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A Study of Self-Organization in Small Systems with Simple Dynamics

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Authors' contributions

This work was carried out in collaboration between both authors. Author WJBO conceived the idea, wrote the program and wrote the first draft. Author JR made significant contributions in editing, organizing and made significant technical suggestions to improve the paper. Both authors made enough contribution to this paper to be listed as co-authors. Both authors read and approved the final manuscript.

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ABSTRACT

Self-organization in small systems of particles with simple dynamic laws has been simulated. The purpose of this work is to investigate self-organization in small systems where we could follow individual particles. The intention is to look for pattern formation as the system evolves. For the two kinds of systems studied, the motion and the final system state for various dynamic iterations are presented. In the first system design, two kinds of particles are simulated. Like particles have a repulsive force, while unlike particles have an attractive force. Initially, the particles are randomly distributed in a two dimensional square bounded region, and then allowed to dynamically interact for a number of iterations. In experiment 1 the particles have different polarity. Using the inverse square law force, modified at short distances, most cases resulted in equilibrium with the particles of opposite polarity paired up. Since this was a state of equilibrium no more movement occurred. In the second experiment, there are two groups of particles initially separated by a boundary. The particles on each side of the boundary are further divided into two groups referred to as strong or weak particles. In this experiment the resulting patterns were clusters of particles. The forces

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among all of the particles can be varied to study the configurations that result from the dynamics. The results of the experiments are presented in graphical format. The main conclusion is that this model can be used to study small dynamic systems.

Keywords: Self-organization; computer simulation; evolution; dynamics; non-equilibrium; particle-to-particle interaction; cluster formation.

1. INTRODUCTION

Self-organization in a system is a naturally occurring phenomenon. It can be observed in a number of different phenomena [1-6]. It can occur in systems with no external or internal control. Only the internal forces guide the evolution of the system. Self-organization can also occur in non-linear dynamical systems. Self-organization has been addressed in many ways. Most of the approaches are statistical-mathematical framework based. One popular approach is referred to as Multi-Agent Systems. This statistical approach is Bayesian based [7]. Another approach is Group Method of Self-Organization Techniques in Modeling. [8].These methods have the advantage of being able to handle a large group of particles or entities and complex dynamics. The work presented here, is a numerical simulation study of self-organization in two different but related systems. The work is presented through the implementation and results of two experiments called Exp1 and Exp2. Exp2 has three labeled parts; 2a,2b and 2c. The emphasis of the study is on systems of simple configuration and the dynamics provided by the simple inverse square force law. Our method has an advantage over the statistical methods of being able to follow individual or small groups of particles throughout the evolution of the system. Our emphasis is on the simplicity of the system, and the resulting outcome using simple dynamics. Our approach has already been used to study the propagation of infectious diseases [9] and to follow the migration across a common boundary [10]. These two papers used the same basic framework as this paper with some modifications. Some of the incidental parts of these papers are repeated here so that this paper clan be as clear and complete as possible. It appears that this approach could be used in any situation in which using a small number of particles is enough to give some insight into the system evolution. It is necessary that the forces can be easily defined and coded.

In Exp1 the system evolves to a frozen and cold state, but may still provide insight into some systems. Exps 2a, 2b and 2c are directed at social scenarios in which two entities share a common boundary. The particle-to-particle interactions allow for particle movement across the border. Thus the results of the movement can be studied and analyzed. Our outlook is that we have created a tool that can be used to study self-organization for a number of different scenarios. The program collects a number of statistics that can be used to study the system evolution.

The systems described here have two kinds or types of particles. Within each type the particles are identical except for initial location. In Experiment 1 (Exp1) the particles are similar, but the two types have different interaction strength. Numerically the strengths are the same but have opposite signs. Additionally, one type of the particles is less responsive to force than the particles of the other type. Hence, with some changes the two types could be designed differently, but the interactions used here within and outside the type are simple to describe. The dynamics are determined by a set of rules that the particles follow. The more responsive particles are referred to as e-particles and have a negative strength, and the less responsive are referred to as p-particles and have a positive strength. In all that follows the system is contained in a square of size 512 by 512. At initiation for Exp1 the particles are located by randomly placing them in the square. A sample initial distribution for a system with 100 particles is shown in Fig. 1. This layout is a two dimensional system, but we believe it could be extended to 3-dimensions fairly easily.

In Exp2 the two types of particles are separated initially by a center line boundary. That is each type is contained in a rectangle of size 256 by 512. One type is on the left side of the square, and the second type is contained on the right half of the square. At the start the particles are randomly located in their respective part of the square. The coordinates of the particles are determined by the use of a random number generator. The particles are sub-divided into strong and weak particles. The strong particles on each side have strength of 25. The weak

particles on each side have strength randomly distributed from 0 to 25. The x coordinates are randomly assigned from 0 to 255 on the left side and 256 to 511 on the right. For both sides, y is randomly distributed from 0 to 511. A sample distribution is shown in Fig. 2.

The dynamics are governed by a set of forces between each pair of particles i and j such that $f_{ij} = S_i * S_j / d_{ij}^2$ where S_i and S_j are the respective strengths of the particles and d_{ij} is the Euclidean distance between the two particles. In all cases considered here the distance between the particles is the Euclidean measure as shown in Fig. 3.

$$dx = x_j - x_i$$
$$dy = y_j - y_i$$
$$d_{ij} = \sqrt{dx^2 + dy^2}$$

Arrow shows direction of repulsive force. All of the others are similar.

The models being described here are similar to but different from the one described by Schilling in 1969 [6,11]. He presented a model of segregation development in racially mixed neighborhoods. His model showed that local interactions can lead to surprising aggregate results, and can lead to the collapse of integrated neighborhoods and to segregated neighborhoods. The model was based on a local decision by a person to be happy or unhappy. A person was declared to be happy if a certain percentage of that person's nearest neighbors were of that person's same race. Otherwise the person was unhappy. An unhappy person was moved. The simulation proceeded by selecting a person and moving that person or not based on the state of happiness. The simulation continued until there were no more unhappy people, and a state of stable equilibrium had been achieved. The results showed that, based on this decision criterion, racial integration collapses. In the work presented here, there is no individual decision, as the particles move under a natural force law. The particles move subject to the applied force to each particle until a state of stable equilibrium occurs, if that is achievable. As discovered during the course of this work the equilibrium state may be a local minimum or a global one. This is consistent with all gradient descent methods. Introducing some low level noise can be used to break out of the local minimum, and allow the system to find another minimum. In some cases, we used this technique to find the global minimum.

The rest of this paper is organized as follows. In Section 2 the method and program are described briefly. Section 3 describes Exp1. Section 4 describes Exp2. Section 5 states the conclusions and potential future work.

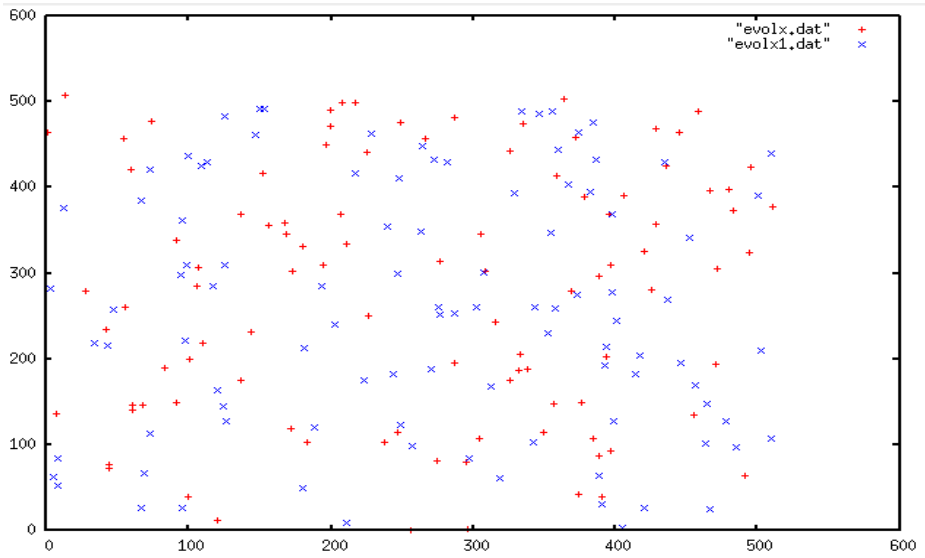

Fig. 1. An initial distribution for Exp.1. 100 particles of e and p. Red "+" denotes e-particles and blue "x" denote p-particles

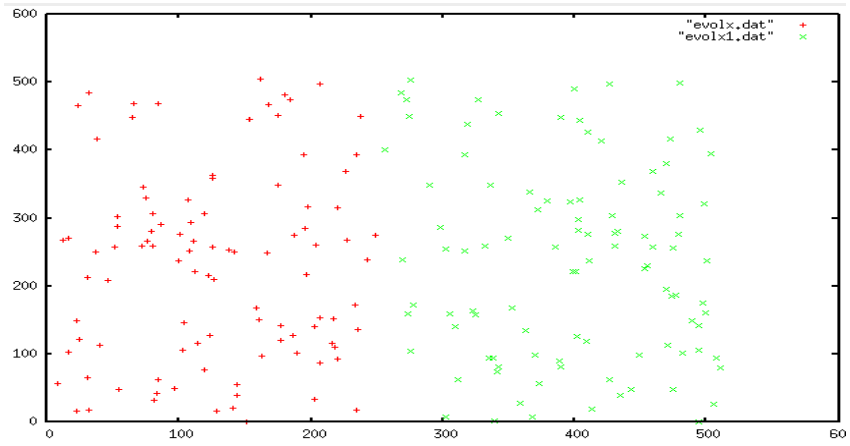

Fig. 2. An initial distribution for Exp2. 100 particles on each side with red "+" denoted on left side and green "x" on right side

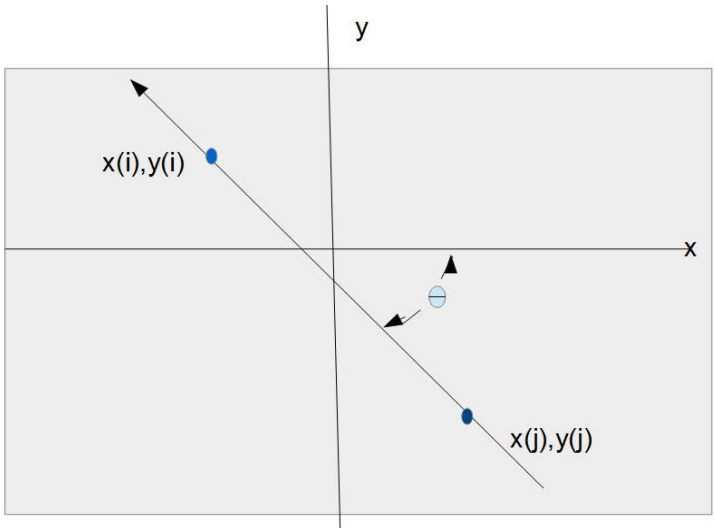

Fig. 3. Graphic description of distance from (x_i, y_i) to (x_j, y_j)

2. METHODS AND MATERIALS

This study is based on computer simulation. The program was specifically created for investigating self-organization of particle systems with the particles interacting through simple forces. The program was written in c using the gnu c compiler under the Debian version of Linux. The program was constructed in modular form so that it could be easily modified for different applications and relatively easy to check out. The various modules are functional in form. There are such functions as "plotty" to handle the plotting function, "distance" to calculate the distance between every pair of particles and of course a function to calculate the forces on each particle. There is a lot of bookkeeping required for the force calculation. The programming is straightforward with no unusual methods used. This approach makes the code easier to understand and modify as needed or desired. Since the distance between every pair of particles was calculated at each iteration step the complexity of the program is at least n^2.

2.1 Experiment 1

In this scenario it is assumed there exists two kinds of particles with simple dynamics and are referred to as e and p particles. The particles are such that every particle interacts with every other particle. Each particle was assigned force

strength. For this experiment the force strength was the same for all particles with value 25. The particles interact such that the force on particle i by particle j was

$$f_{ij} = S_i S_j / d_{ij}^2 , \quad (1)$$

where S_i and S_j are strengths of particles i and j and d_{ij}^2 is the square of the distance between particle i and particle j. The force is either repulsive or attractive and acts along the line between the two particles. If e represents one type of particle and p represents the other type then the e-e and p-p interactions are repulsive. The e-p and p-e interactions are attractive. The simulation proceeds at discrete intervals according to the dynamics equation $d = d_0 + f\,\delta t$ where d_0 was the x or y coordinate before an iteration step, d was the coordinate after an iteration step, δt was the size of the iteration step, which was either δx or δy depending on whether x or y was being updated, and *f* was the force acting on the particle in x or y directions. The p particles were assumed to be less responsive to a force than the e particles. This was implemented by making the response to the applied force a number of times smaller for the particles than for the e particles. The δt for e particles was 0.01 and for the p particles was usually 100 to 1000 times smaller. The simulation proceeds as follows:

a) n particles of each type were created and placed at random in a 512 by 512 square.
b) The strength of each particle was assigned. All of the particles have the same strength of 25 and is either positive or negative.
c) The vector distance between every pair of particles was calculated.
d) The net force on each particle (Eq1) was determined by vectorially adding all of the forces from the other particles. See Fig. 3.
e) The dynamics were then applied to obtain the new position of each particle.
f) With the new locations known the process starting at step c was iterated. The process is repeated for a given number of iterations so that the results can be studied.

In looking for a set of operating parameters, we found that a strength of 255 for each particle was too large. The original strength S and δt were too large. The effect of these choices was that a particle moved too far in during an iteration, so that the particles were hitting the edge of the square too soon and too often. When a particle left the square it was re-injected to keep the number of particles constant. The rule in use was that if a particle left the square on an iteration it was injected back into the square at a random location. If this happens too frequently the effect was that the chance of reaching equilibrium was unlikely. It was found a strength, *S* of 25 to be usable and δx and δy were both 0.01 for e particles and 0.001 for p particles. Other values could be chosen with successful results.

The locations of both e and p particles was known and could be graphed. The final state graph appears to show that pairs of particles are coincident. This is shown in Table 1 that for a 16 particle configuration the coordinates of a pair of e-p particles was the same to at least four significant figures. The particles are regarded to be very light or even of zero mass. That is they are light enough as to have no kinetic energy. The potential energy *V* then is the only energy available. The potential energy by one particle of a pair contributed by a pair is

$$V = \pm S_i S_j / d_{ij} . \quad (2)$$

So at initiation we have a certain potential energy. The potential energy for a pair of particles varied based on the distance between the particles. The strength of each particle was 25. Using Eq.2 for two particles at a distance of 10^{-4} the potential energy was 6.25×10^6. If two particles were at opposite corners of the square the potential, again using Eq.2, contributed by one is $625/512\sqrt{2}$. The final configuration in most cases was such that the particles are coincident pairs. Hence, at this point there are no unbalanced forces and the potential energy is zero. At the beginning the particles are uniformly distributed so the entropy is large.

The implementation of the dynamics deserves some further explanation. Employing these dynamics bears a significant similarity to integrating the equations of motion for the inverse square law force. When two particles are far apart the force is small because of the distance squared in the denominator. Hence, when the distance to be moved by the particles is small a relatively large step size could be used. As the particles get closer the force naturally increases. At certain distance between an e-p pair, called the critical distance, d_c, the distance

to be moved was equal to the distance of separation. This distance was

$$d_c^3 = S_i S_j \, \delta t \, . \tag{3}$$

If the distance between the particles was smaller than dc the two particles would over shoot each other when moved via the dynamics. It was necessary to shorten the step size to prevent this. This was accomplished by moving the particle 0.1 of d, the distance between the particles. The distance was calculated and the particles were moved as appropriate.

The program was run many times for 2,10,16 and 100 particles of each type. We ran the 2 particle case as well as the 10 particle cases to gain confidence that the program was correct. In the two particle case there were two particles of each type, i.e. two e particles and two p particles. There was convergence to two pairs. With the parameters used here the number of iterations to convergence was about 3 million. The result was a pairing of the particles with one e and one p particle in each cluster. All of the cases ended in a 1-1 pairing. The potential energy was at a minimum and there was no movement by an e or p particle. Further, any try at movement by a particle would be corrected on the next iteration by the attractive forces between the e and p particles. After convergence a number of iterations confirmed no further movement takes place. The results for the 10 particles show a one-to-one pairing of e and p particles after about 5 million iterations. Also the e and p particles were coincident at the final stage for the 16 particle case. Since the results for the 10 particle, 16 particle and 100 particle models are very similar most of the discussion will be on the 100 particle case. However, included are results from one experiment for 16 particles. The number of iterations required for convergence in all cases varies depending on the initial conditions. The final results for the 16 particle case showed a one-to-one pairing after about 10 million iterations. The numerical values of the e and particle coordinates are given in Table 1. Fig. 4 shows the initial configuration, and Fig. 5 shows the particles after full convergence to a one-to-one pairing.

Table 1 demonstrates the pairing of the particles after 10 million iterations. In all of these cases once the one-to-one pairing is found no further movement occurs even after many more iterations, and the potential energy remains constant. For the 100 particles of a type case we are showing a number of configurations through the evolution. Even after a large number of iterations a one-to-one pairing does not occur in all cases with the parameters used. Some of the p particles have captured two e particles leaving some of the e particles unpaired. The initial configuration is shown in Fig. 6. Some of the p-particles have captured two e particles leaving some of the e particles unpaired. Results are shown in Fig. 7. The configuration was stuck in a local minimum. Further changes were impossible unless something occurred to break the deadlock. That this can occur can be seen by considering two e particles on one p particle. If one of the e particles tries to move that movement would be counteracted on the next iteration. Depending on the exact configuration here the net force from the two nearby particles tends to cancel out. However, all real systems have some inherent noise. To break the deadlock a test was used to find if two e particles are within a check distance of 0.001. If this test was passed the two e particles were moved a random distance and the iterations continued. The distance moved was randomly distributed over -1.0 to 1.0. This was sufficient to break the deadlock. With this the system was free to seek another minimum in the potential energy. Introduction of the noise as needed allows the system to evolve to a pairing of all e and p particles. The results are shown in Fig. 8 and the final coordinates of the particles are in Table 2.

The collection of two e-particles by one p-particle is interesting though. This was some evidence of a collection of particles occurring. However, as mentioned above this configuration was a stable equilibrium, and after many more iterations the potential energy remains unchanged the particle coordinates remained the same to 6 significant figures. It is possible that larger clusters could form and we will look at that in Exp2. These results involving an equilibrium condition deserves some discussion in light of Earnshaw's theorem. Earnshaw's theorem says that a set of discrete charges cannot exist in equilibrium in the inverse square law field. However, here at the end the e and p particles are paired, and there is no force field as all charges are net zero. This can happen because in this work the inverse square law dynamics are modified at close range. If this was not done the particles would overshoot and the dynamics would continue. So this modification negates the assumption of the theorem so that an equilibrium condition can be achieved. This system evolved to a cold and frozen system, and it is not very interesting from

that point of view. However, there may be situations where equilibrium is a desired outcome. One possible situation where such conclusion could be of interest is animals looking for a mate or humans looking for a spouse. The underlying dynamics simply provided some movement to the process. Of course, most of the time in mate searching male-to-male interaction is negative as is female-to-female. However male-to-female and female-to-male interaction is attractive. During the movement the attractive particles can approach each other and eventually pair up. This could lead to a stable social environment.

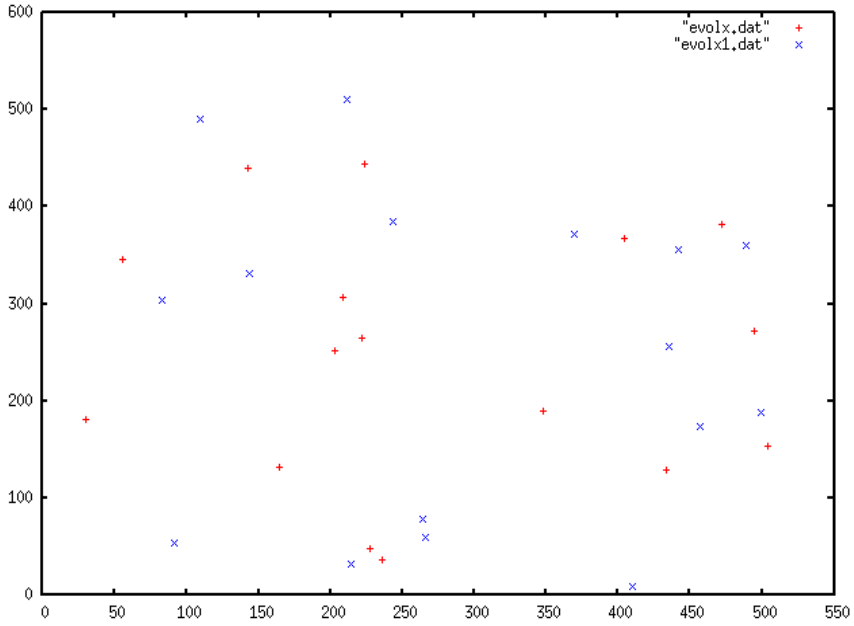

Fig. 4. Exp.1 Initial distribution of 16 particles with red "+" denoting e-particles and blue "x" denoting p-particles

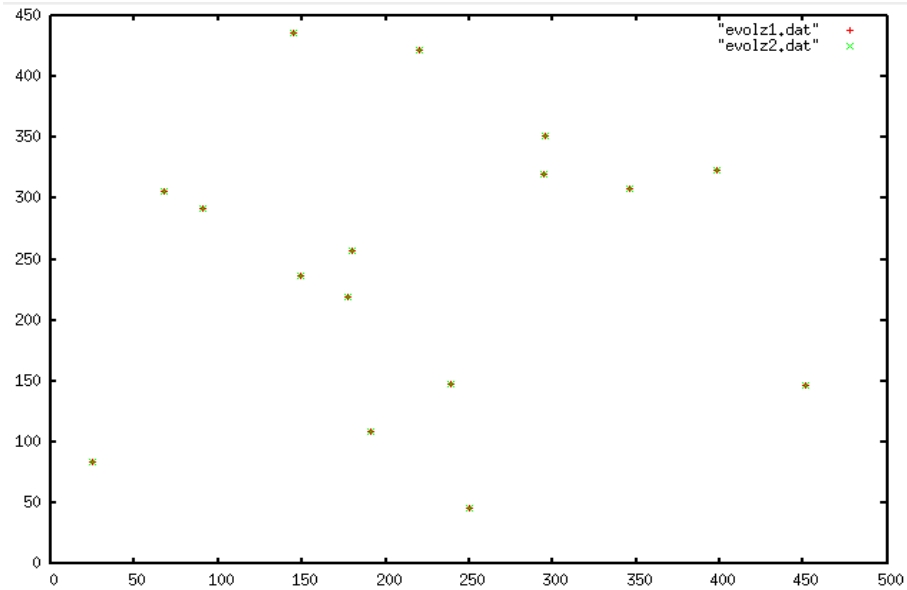

Fig. 5. Exp1 Final distribution of 16 particles after 10 million iterations showing full pairing.

Table 1. Final coordinates for e and p-particles 16 particles 10 million iterations Columns 1 and 2 are for the e-particles and columns 3 and 4 are for p-particles

Particle	e_x	e_y	p_x	p_y
1	90.9	291.9	90.9	291.9
2	177.6	219.0	177.6	219.0
3	250.2	46.3	250.2	46.3
4	238.9	147.8	238.9	147.8
5	451.7	146.2	451.7	146.2
6	179.6	257.4	179.6	257.4
7	67.6	305.6	67.6	305.6
8	149.0	237.0	149.0	237.0
9	295.9	351.1	295.9	351.1
10	345.8	307.6	345.8	307.6
11	190.9	108.4	190.9	108.4
12	25.0	83.5	25.0	83.5
13	219.7	421.3	219.7	421.3
14	398.2	322.8	398.2	322.8
15	294.7	319.6	294.7	319.6
16	144.9	435.9	144.9	435.9

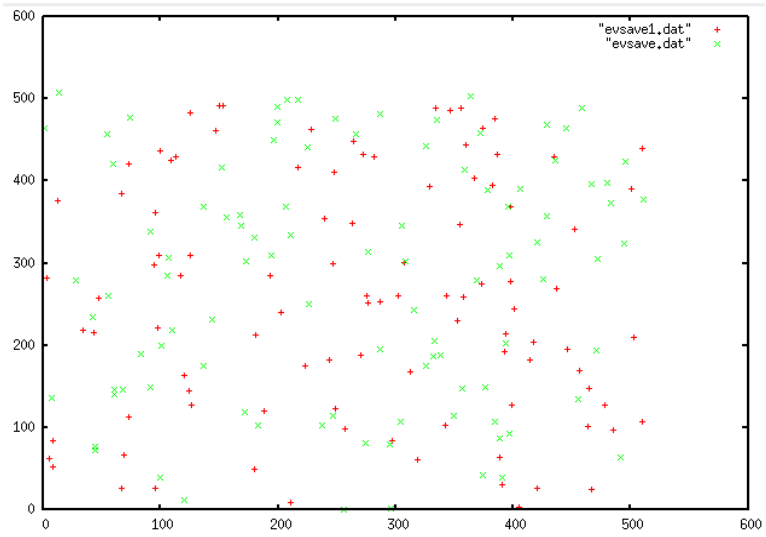

Fig. 6. Exp1 Initial distribution for 100 particles with red "+" denoting e-particles and green "x" denoting p-particles

2.2 Experiment 2a, 2b, and 2c

These experiments were performed as described earlier. The interactions can be set between the strong and weak particles can be selected to be attractive or repulsive. Therefore, the interactions can be set to reflect a number of different interesting scenarios. The idea is that by modifying the type of force (attractive or repulsive), the particle strength, and the number of weak and strong particles a number of scenarios can be simulated. However, we have investigated only a couple. Consider two countries divided by a boundary. Each entity, country or state had its own government or rules and population. It was assumed that on each side of the boundary strong and weak forces are attractive within their respective group. The

particles support within their group. It is assumed that the weak particles dislike the governance on left side of the boundary. Right side weak were favorable to the governance. Across the boundary all of the forces were repulsive except that the weak are sympathetic within their own group and with those on the opposite side. Using s to represent a strong particle and w to represent a weak particle the interactions are shown in Fig. 9.

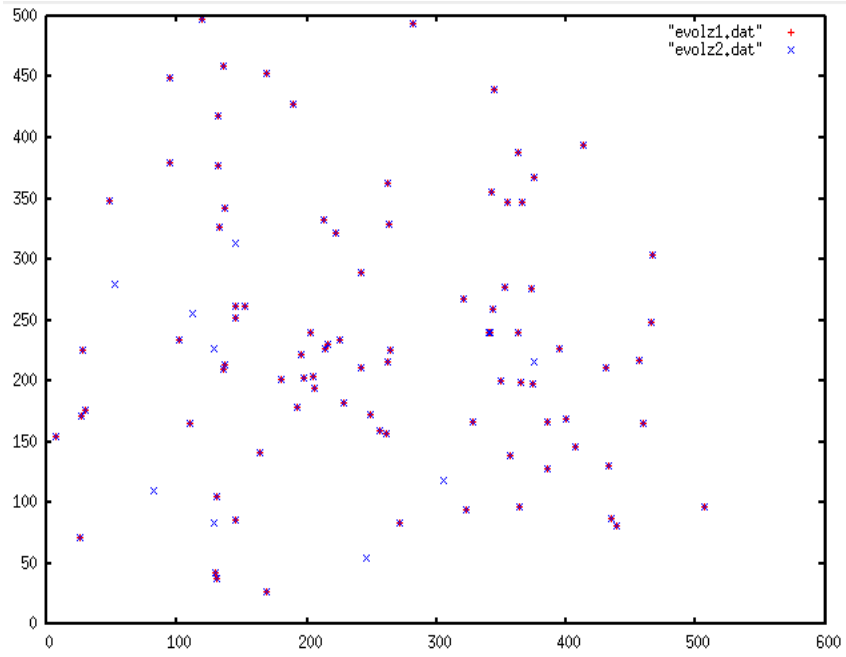

Fig. 7. Exp.1 100 particles after 30 million iterations shows 9 unpaired particles

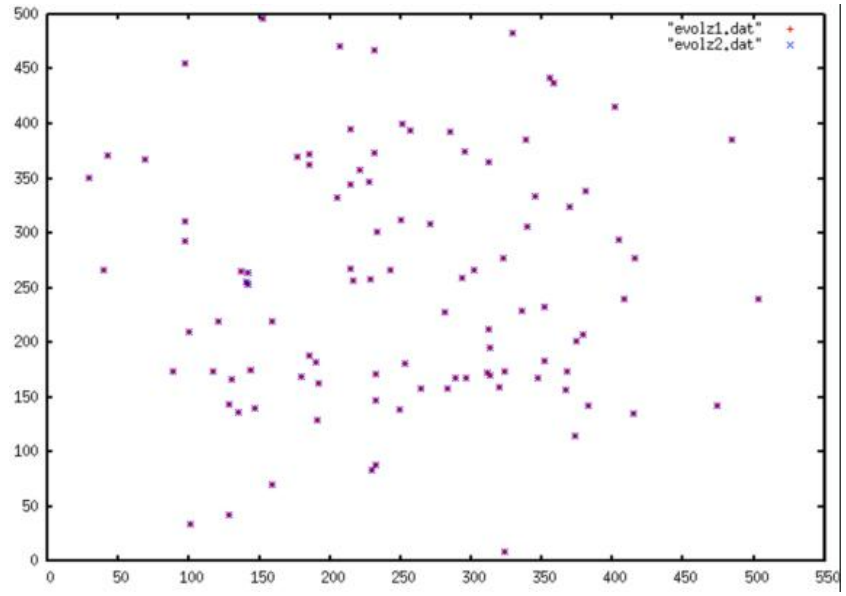

Fig. 8. Exp1 100 particles after 30 million iterations with noise added showing full pairing

Table 2. Final coordinates for 100 e and p-particles 30 million iterations with noise added showing full pairing occurs

Particle	e_x	e_y	p_x	p_y	Particle	e_x	e_y	p_x	p_y
1	484.3	385.5	484.3	385.5	51	226.6	346.6	226.6	346.6
2	97.1	455.8	97.1	455.8	52	369.7	324.3	369.7	324.3
3	68.4	367.1	68.4	367.1	53	380.9	339.0	380.9	339.0
4	339.6	306.5	339.6	306.5	54	146.4	139.4	146.4	139.4
5	312.1	212.7	312.1	212.7	55	128.0	144.1	128.0	144.1
6	295.1	375.2	295.1	375.2	56	231.7	88.1	231.7	88.1
7	415.4	276.8	415.4	276.8	57	140.2	256.0	140.2	256.0
8	323.4	9.3	323.4	9.3	58	136.6	265.0	136.6	265.0
9	184.9	363.0	184.9	363.0	59	134.4	136.9	134.4	136.9
10	101.0	34.3	101.0	34.3	60	248.7	139.4	248.7	139.4
11	312.9	195.3	312.9	195.3	61	338.5	385.3	338.5	385.3
12	414.8	135.6	414.8	135.6	62	178.7	169.9	178.7	169.9
13	269.9	308.9	269.9	308.9	63	190.7	128.7	190.7	128.7
14	344.7	333.5	344.7	333.5	64	191.6	162.5	191.6	162.5
15	216.0	256.3	216.0	256.3	65	231.8	147.9	231.8	147.9
16	213.7	395.1	213.7	395.1	66	97.1	292.8	97.1	292.8
17	143.1	174.9	143.1	174.9	67	128.4	42.6	128.4	42.6
18	229.3	83.7	229.3	83.7	68	157.9	219.8	157.9	219.8
19	288.1	167.2	288.1	167.2	69	284.2	392.1	284.2	392.1
20	404.2	294.5	404.2	294.5	70	214.3	345.2	214.3	345.2
21	250.7	399.8	250.7	399.8	71	351.5	233.1	351.5	233.1
22	374.2	201.2	374.2	201.2	72	401.7	415.6	401.7	415.6
23	204.7	333.1	204.7	333.1	73	407.9	240.6	407.9	240.6
24	39.1	265.9	39.1	265.9	74	242.2	265.9	242.2	265.9
25	227.7	258.4	227.7	258.4	75	280.8	228.0	280.8	228.0
26	328.6	483.4	328.6	483.4	76	313.1	170.2	313.1	170.2
27	366.5	157.2	366.5	157.2	77	230.7	467.7	230.7	467.7
28	323.0	174.0	323.0	174.0	78	99.8	209.9	99.8	209.9
29	42.2	371.4	42.2	371.4	79	312.2	364.8	312.2	364.8
30	373.0	114.7	373.0	114.7	80	117.2	174.2	117.2	174.2
31	189.6	182.5	189.6	182.5	81	206.0	470.6	206.0	470.6
32	29.4	350.2	29.4	350.2	82	351.5	184.0	351.5	184.0
33	382.3	143.1	382.3	143.1	83	213.8	267.1	213.8	267.1
34	176.4	369.7	176.4	369.7	84	129.9	166.3	129.9	166.3
35	158.7	70.8	158.7	70.8	85	322.2	277.4	322.2	277.4
36	358.0	437.7	358.0	437.7	86	301.9	166.3	301.9	166.3
37	503.6	239.6	503.6	239.6	87	354.9	441.7	354.9	441.7
38	249.3	312.5	249.3	312.5	88	141.2	253.3	141.2	253.3
39	141.5	263.7	141.5	263.7	89	231.9	171.4	231.9	171.4
40	252.5	181.0	252.5	181.0	90	120.5	219.7	120.5	219.7
41	295.4	168.2	295.4	168.2	91	335.1	229.2	335.1	229.2
42	151.5	496.7	151.5	496.7	92	184.8	372.8	184.8	372.8
43	96.7	310.8	96.7	310.8	93	255.9	394.2	255.9	394.2
44	474.3	143.0	474.3	143.0	94	88.3	173.7	88.3	173.7
45	367.8	173.6	367.8	173.6	95	264.3	158.8	264.3	158.8
46	246.7	168.3	246.7	168.3	96	230.9	373.6	230.9	373.6
47	311.1	172.1	311.1	172.1	97	293.1	259.7	293.1	259.7
48	232.3	301.5	232.3	301.5	98	319.1	159.4	319.1	159.4
49	283.1	158.5	283.1	158.5	99	220.4	357.5	220.4	357.5
50	379.2	207.2	379.2	207.2	100	184.2	188.4	184.2	188.4

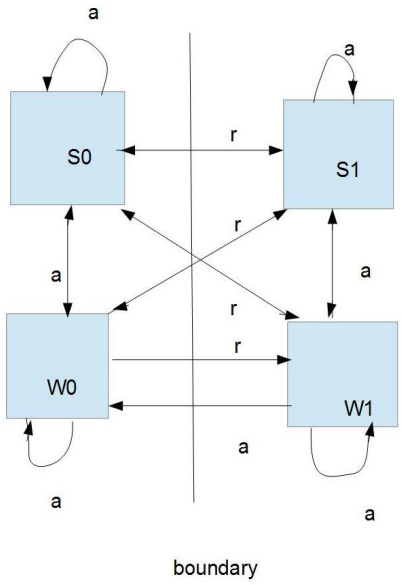

Fig. 9. Describes the forces between the particles for Exp2a. S represents strong particles, W represents weak particles, "a" represents an attractive force and "r" represents a repelling force. Double headed arrows indicate the force is both ways

Table 3. A List of the force interactions defined for Exp2a, 2b, and 2c, the letter "a" is for attractive and "r" is for repel

Force number.	force label	Exp1,2	Exp3,4
1	w0 to w0	a	a
	w1 to w1	a	a
2	w0 to s0	r	r
	w1 to s1	a	a
3	s0 to w0	r	r
	s1 to w1	a	a
4	s0 to s0	a	a
	s1 to s1	a	a
5	w1 to w0	r	a all others same
	w0 to w1	r	a
6	s1 to s0	r	r
	s0 to s1	r	r
7	s1 to w0	a	a
	s0 to w1	r	r
8	w0 to s1	a	a
	w1 to s0	a	r

Here, s refers to strong particles, w refers to weak particles, 0 refers to the left side of the square and 1 refers to the right side of the square. The sign of the each force is given in an input file. If the input is plus one the force is attractive. If the input is minus one then the force is repelling. The signs are just the opposite for the potential. The forces are as listed in Table 3, which is a repeat of Fig. 9. The strong particles were fewer in number than the weak particles. This corresponds to a situation where the leaders or elite were strong and the general population was weak. Here, there are 20% strong particles and 80% weak particles on each side. Once given the initial locations, the net force on each particle was calculated and the dynamics applied. The same simulation steps were employed as for Exp1. The simulation process was continued for a number of pre-assigned iterations or until stable equilibrium was achieved. The final

particle locations were saved so the simulation could be restarted for another set of iterations until equilibrium was reached, if possible. Various parameters are calculated and saved for further analysis of the system evolution. The results will be described later. This model could be used to model different scenarios my modifying the way the particle forces are described as repulsive or attractive. Exp 2b modified the weak particles on left side to repel weak particles on right side as shown in Table 3. This may be thought of as a jealousy factor, but not one of hatred. An initial distribution for Exp 2b is shown in Fig. 10. For Exp 2b not much changed after 10, or even 20 million iterations. Hence, only the 30 million iteration case is discussed. At this point the potential energy had not settled down to a constant value. This appears to be due to the re-injection method used. The re-injection method was that if a particle left the square it was placed back in the square at a random location. Results from Exp. 2a are shown in Fig. 10-17. These figures start with the layout for 100 particles, and continue through diagrams for 30 million iterations.

Various statistics were calculated during a run. The shortest distance between particles was calculated, as was the average of the shortest distances, the variance, the standard deviation and the distribution function of the shortest distances. The average distance moved, the potential energy and the number of particles moved across the boundary were also measured and some of these will be discussed. To follow the evolution of the system particle clusters were calculated. The clusters for particles of each kind are calculated after a specified number of iterations. The cluster calculation proceeded as follows. A point in the 512 by 512 grid was selected starting in the upper left hand corner, and then the points were selected proceeding horizontally and then vertically until all points in the grid are processed. When a point was examined it was added to the cluster if it has n_o neighbors closer than r_o, a radius around the selected point. Let n_o be the number of particles within radius r_o. A radius of r_o = 40 and n_o = 3 has been used often, but other values were also used. Both of these parameters are input parameters and are easily adjusted. An additional program also exists that can be run after the fact for investigation of the clusters for different values of r_o and n_o. However, there was a big surprise in the movement data as shown in the figures. We expected the movement to be the greatest from left-to-right that is from the poor to the rich side. However in some cases the migration from right-to-left was the larger.

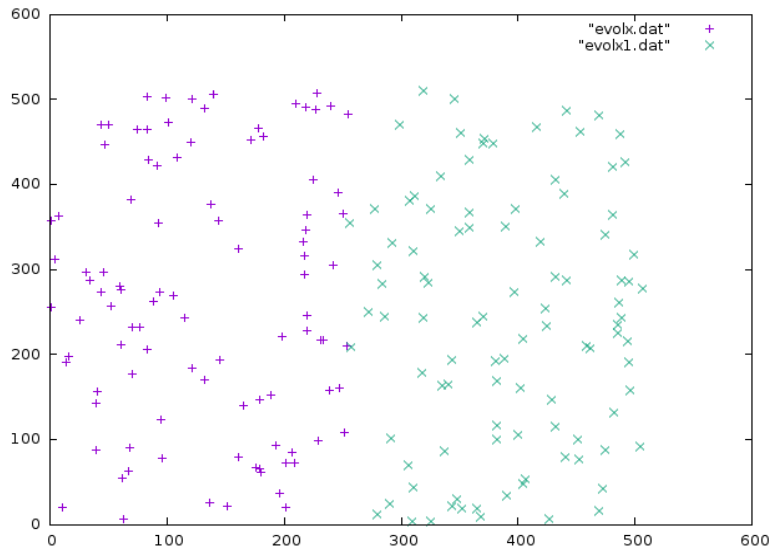

Fig. 10. Initial distribution for Exp.2a. 100 particles on each side, red "+" for left side particles and green "x" for right side particles

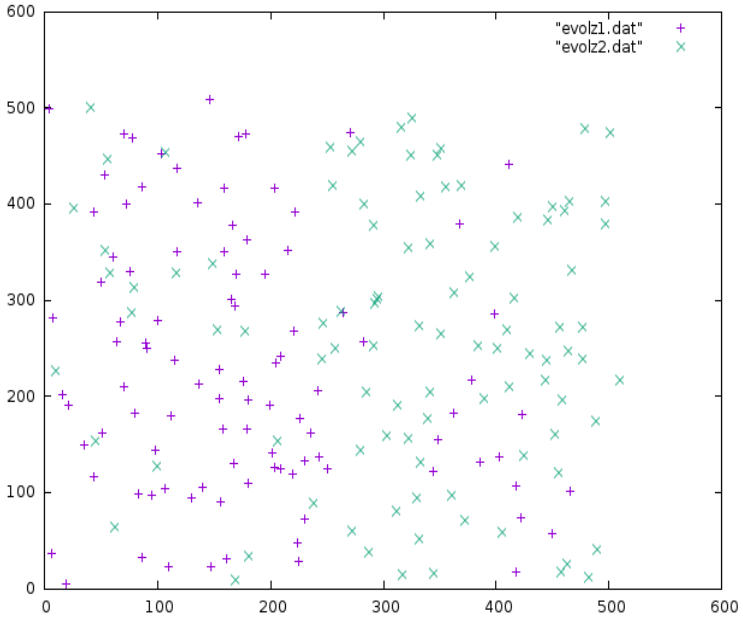

Fig. 11. Exp.2a Distribution after 10 million iterations

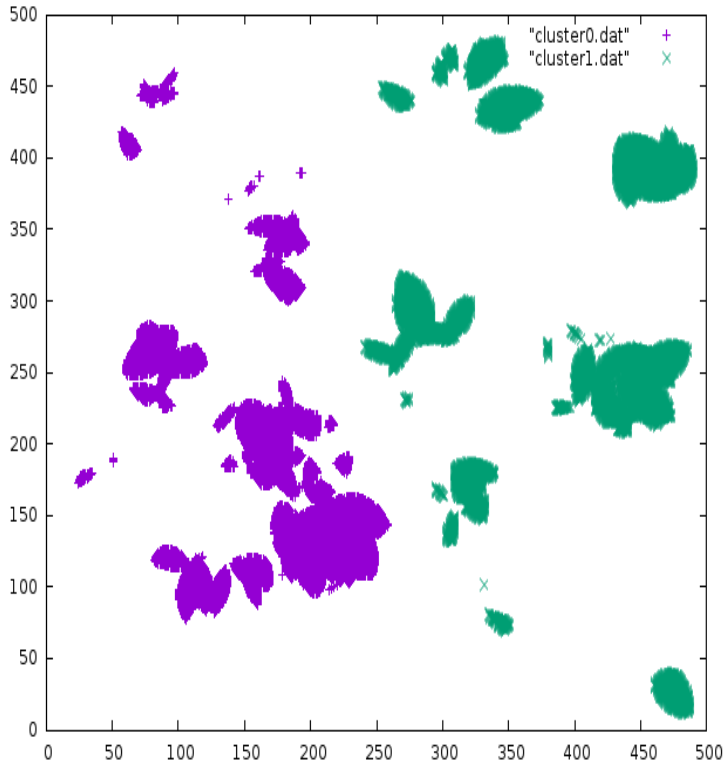

Fig. 12. Exp. 2a Cluster diagram after 10 million iterations, purple "+" for left side particles and green "x" for right side particles. Showing minor clustering across boundary

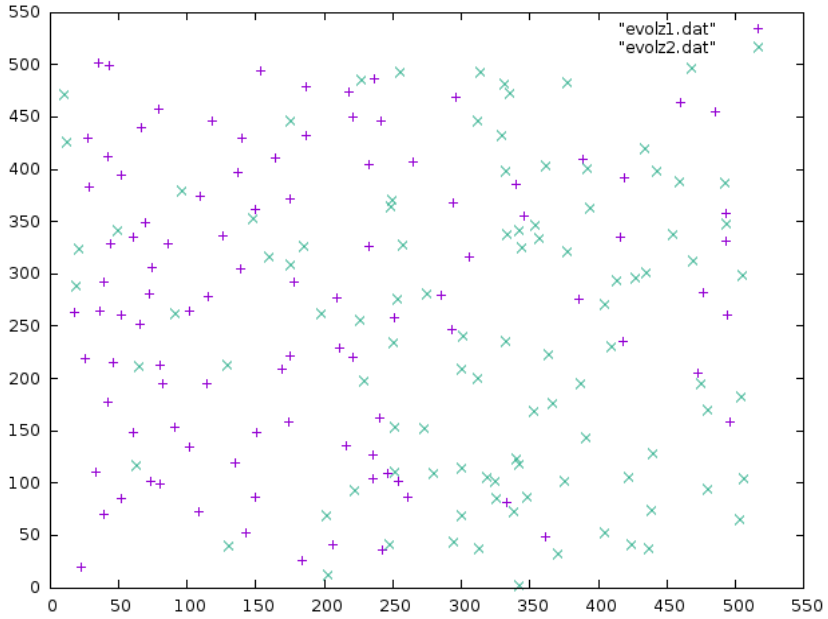

Fig. 13. Exp2a Distribution after 20 million iterations, red "+" for left side particles and green "x" for right side particles and showing some boundary crossing. Migration left to right 24 particles, right to left 31

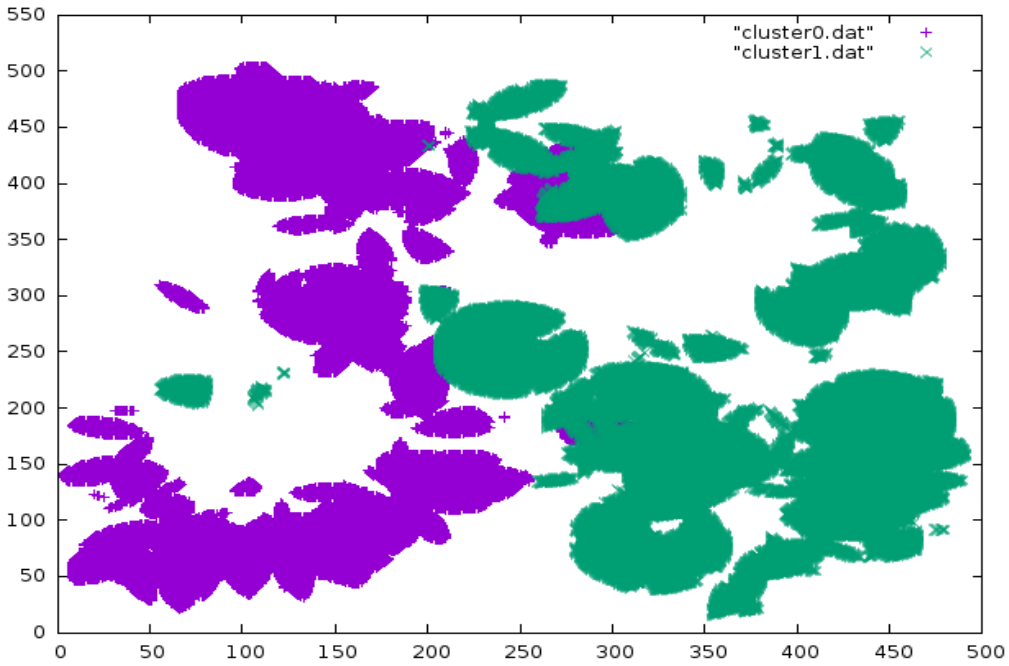

Fig. 14. Exp. 2a Cluster diagram after 20 million iterations, minor clustering resulting

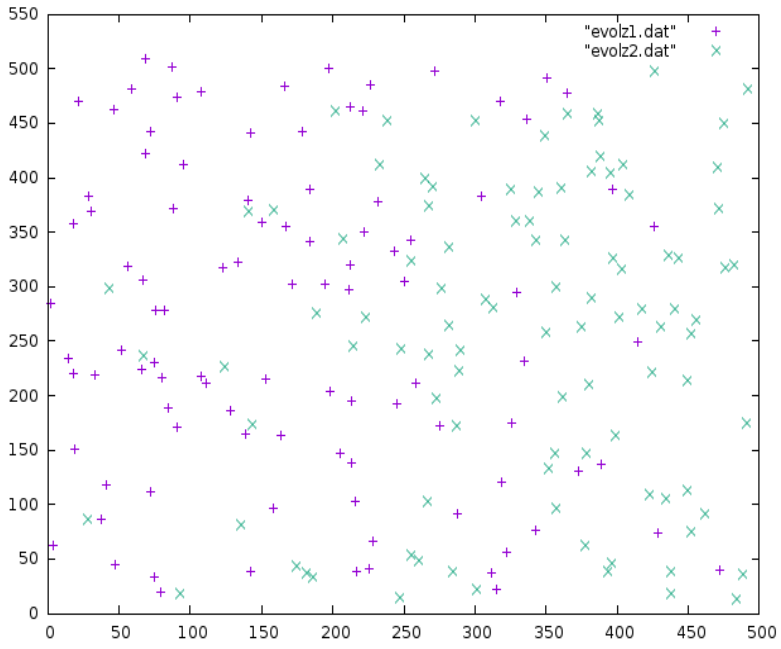

Fig. 15. Exp.2a Distribution after 30 million iterations, red "+" for left side particles and green "x" for right side particles. A good number crossing border 24 left to right 23 right to left

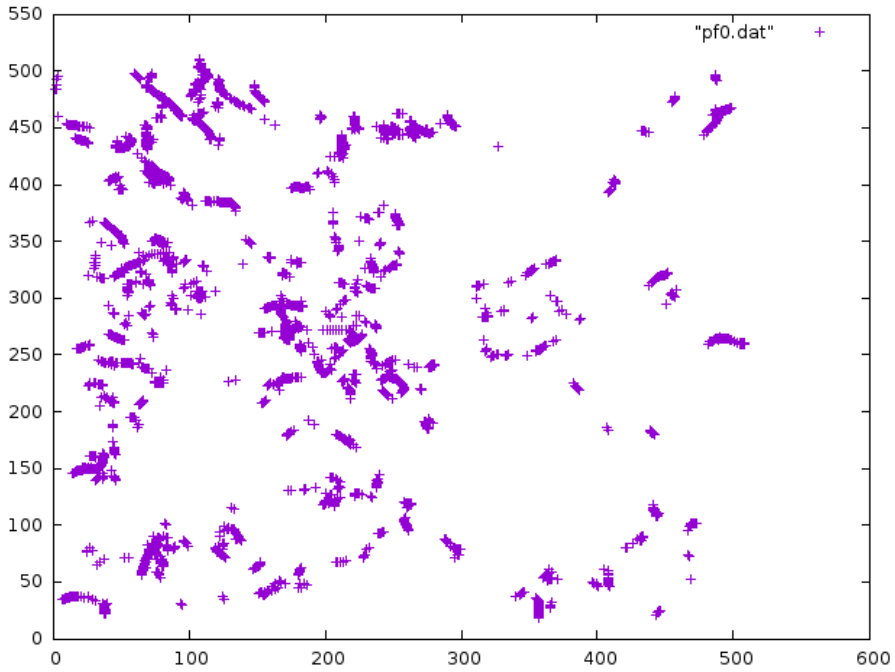

Fig. 16. Exp2a Movement of a strong particle # 10 on left side. All of the others are similar in nature

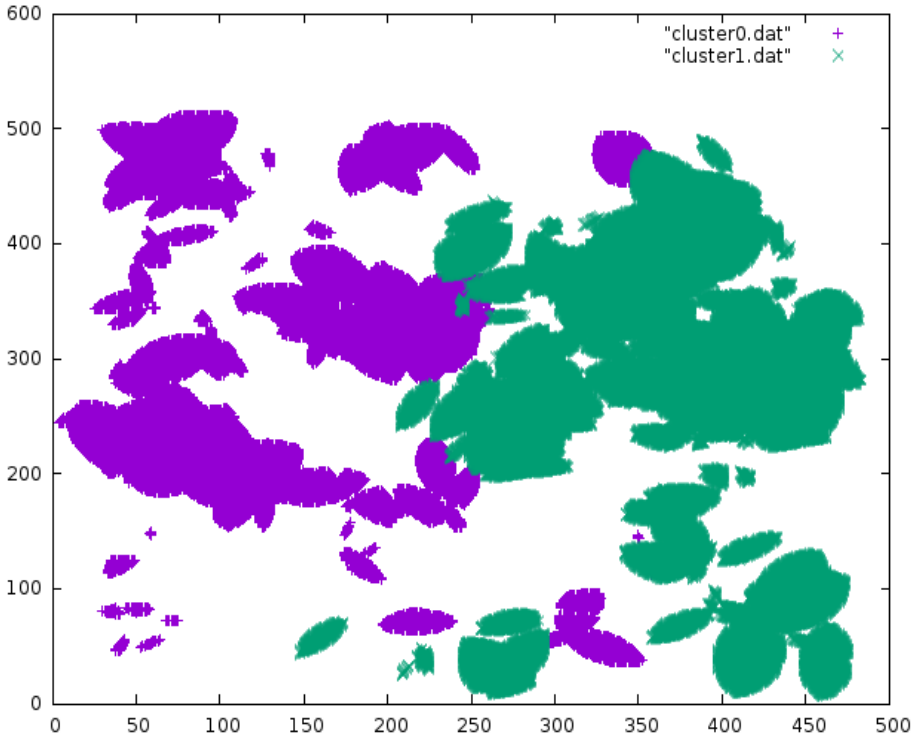

Fig. 17. Exp2a Cluster diagram 30 million iterations, purple for left side particles and green for right side particles Good clustering on each side

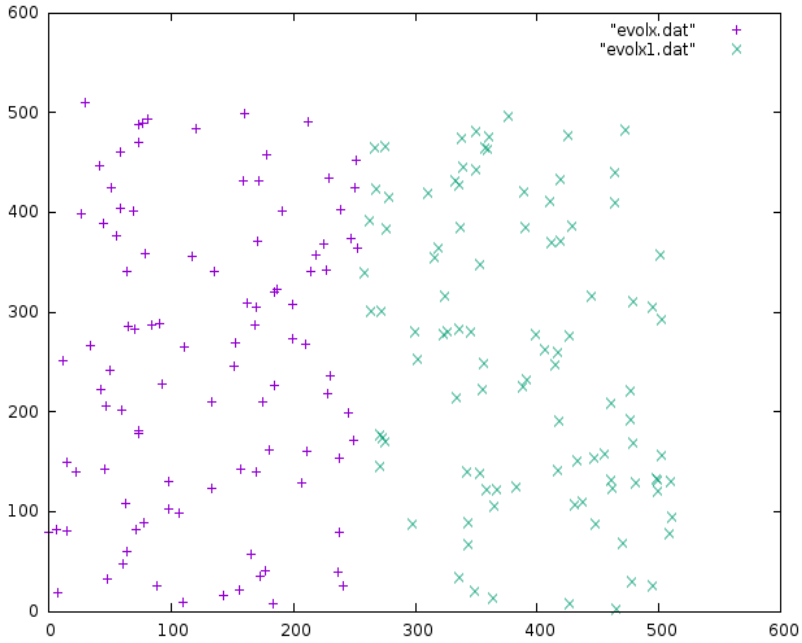

Fig. 18. Exp2b 100 particles 30 million iterations initial configuration, purple "+" for left side and green "x" for right side

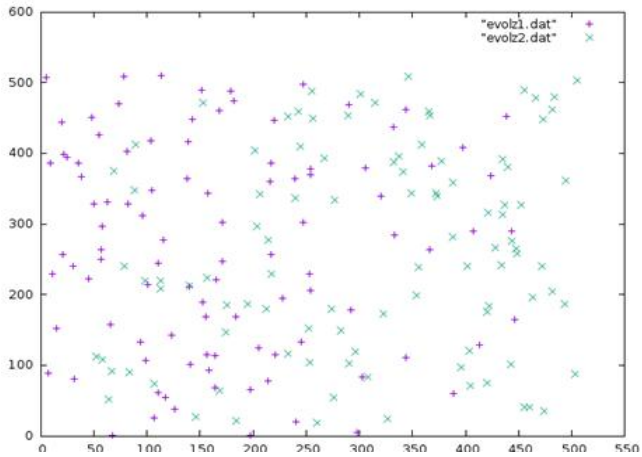

Fig. 19. Exp2b Final configuration 100 particles after 30m iterations purple "+" for left side and green "x" for right side

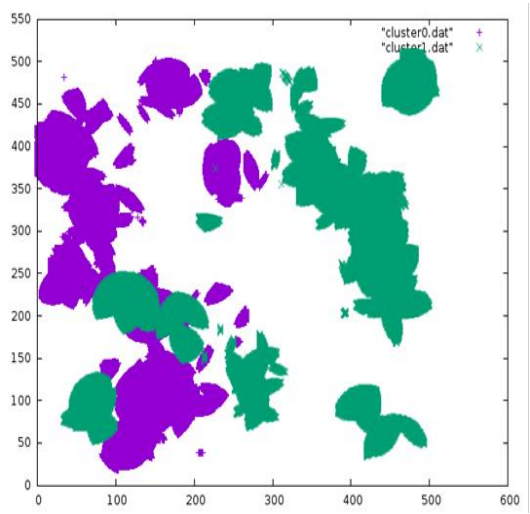

Fig. 20. Exp2b 100 particles 30m iterations Cluster diagram. Left to right migration 20. Right to left migration 36

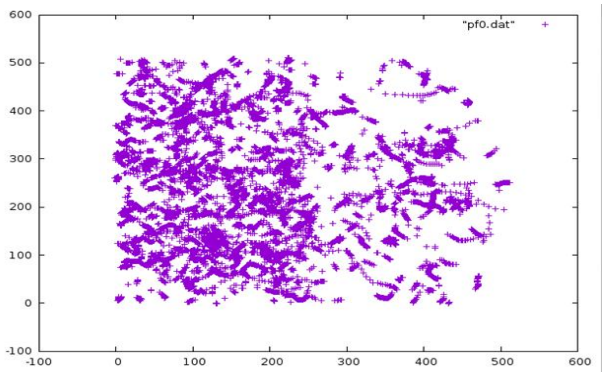

Fig. 21. Exp2b 100 particles 30 million iterations. The movement of particle10 from the left side. It can be seen that the particle spends time on both sides so the count crossing is only taken at a snapshot

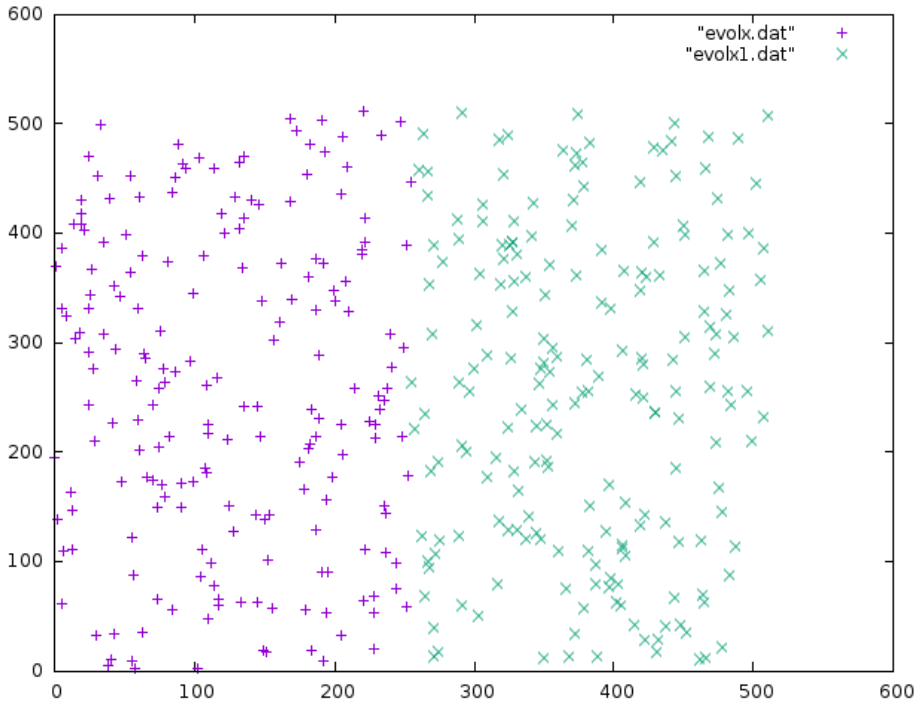

Fig. 22. Initial distribution for Exp2c 200 particles, purple "+" for left side and green "x" for right side

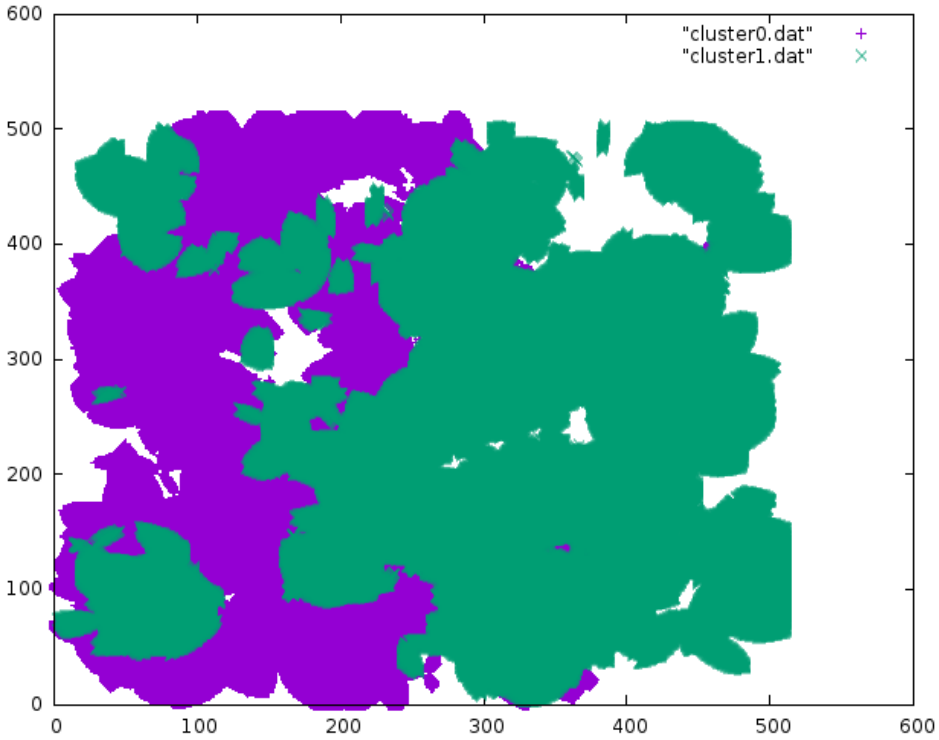

Fig. 23. Cluster diagram Exp2c 200 particles after 30 million iterations

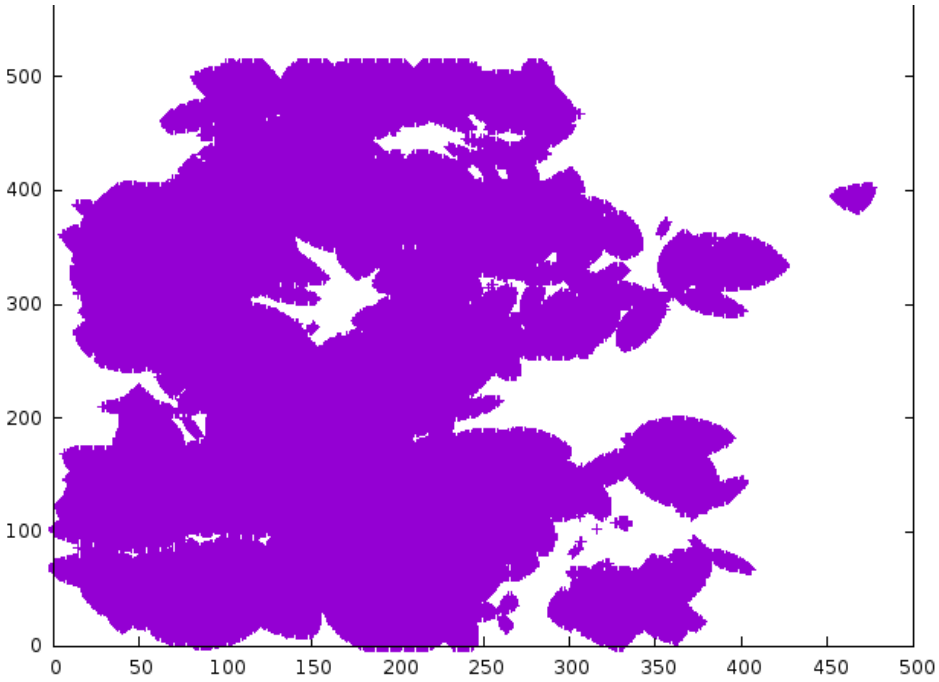

Fig. 24. Cluster left side 200 particles after 30 million iterations

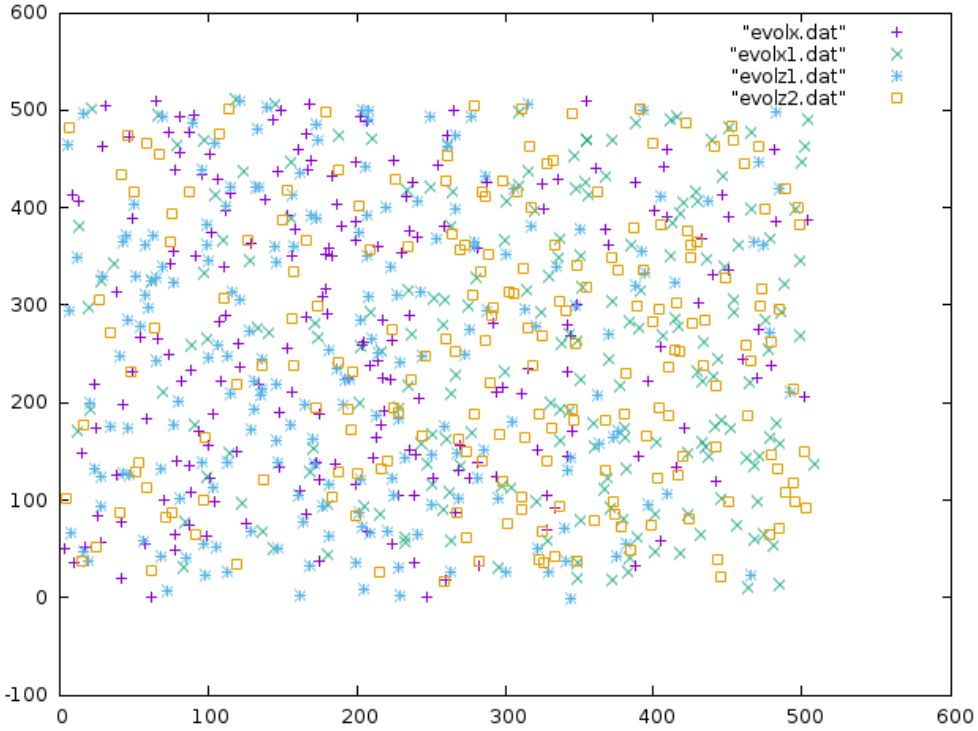

Fig. 25 Initial and final distributions Exp.2c 200 particles after 30 million Iterations, red "+" for initial left side and green "x" for initial right side, blue "*" for final left side and orange box for final right side. Left to right side cross 59. Right side to left side cross 66

Exp 2b was the same as Exp2a with one force changed as shown in the Table 3. Results are shown starting with 100k iterations through 30 million in Fig.18-21. Fig. 18 displays the initial distribution, while Fig. 19 shows the distribution after 30 million iterations. The cluster diagram is shown in Fig. 20. It appears that the movement was not as great as might be expected. The movement across the border was 20 particles for left-to-right and 36 particles for right-to-left. It is surprising that the right-to-left movement is greater than left-to-right movement. It appears that changing the one force did not have any appreciable effect on the outcome. This outcome is contrary to our intuition that a bad government and poor living conditions would lead to mass movement. Fig. 21 shows the movement of particle 10 from the left side during the evolution. This particle spends significant time on both sides of the border.

Exp.2c was the same as Exp2b except it had two hundred particles per side. These results are shown in Fig. 22-25. The last results are for 30 million iterations and the cluster formations are shown in Fig. 23 and 24. We included Fig. 24 because of the overlap of the clusters in Fig. 23. Fig. 25 displays the final distribution after 30 million iterations. The movement was 59 particles from left-to-right and 66 right-to-left. Again we got an unexpected result.

3. DISCUSSION, CONCLUSIONS AND FUTURE WORK

Our objective in this work was to explore self-organization using systems of few particles and simple dynamics. We have presented the results of two experiments to demonstrate this approach. In Exp.1 we had two kinds of particles that interacted through the inverse square law.

Starting form a disordered state the system ended in a totally ordered system. We did notice that as the number of particles increased the likelihood of the system getting caught in a local minimum increased. This is a common circumstance in gradient descent methods. Often times the system can be freed from the local minimum by adding some low level noise. We used this technique here and in those cases the addition of the noise allowed the system to find the global minimum. In Exp.2 the system consisted of two entities separated by a border. A set of forces were defined between the particles on the sane side of the square and on opposite sides. Then the dynamics were applied. In Exp.2 did we not see a minimum occur. Some patterns in the form of clusters were observed. These clusters came and went. One of the interesting findings was that even though one side was defined to be poorer with worse living conditions than the other side, the particles did not show any real preference for either side. This is evidenced by the count of the number of crossing of the border by particles from either side.

We have refrained from likening these particles to any real physical particles. It is just a fun exercise to see how a system can evolve from total disorganization to complete order in a system simple enough to follow the dynamics. Therefore we might look at some comparative parameters of physical systems. For instance, the density of particles in these experiments is very low. The density per unit area is $200/512x512=7.6x10^{-4}$ particles/unit area. A mole of hydrogen gas at standard temperature and pressure is $6.02x10^{23}/cm^3$. In the ionosphere the maximum density of electrons is about $10^6/cm^3$. Second, the time span is very short. There are 31536000 seconds in a year. If we regard the step interval used here to be 10^{-4} then 10^6 iterations corresponds to about 100 seconds and $20x10^6$ iterations corresponds to 33 minutes. However, if we regard the step interval here as the fundamental time unit, a second, then a million iterations corresponds to about 11 days. These are indeed short intervals of time. Using 16 million iterations corresponds to about 0.5 year.

This paper only reports results for a small number of configurations and a small number of parameters. The forces can be adjusted as well as to which particles interact with each other. Presented here is a program or simulation to describe the migration of particles under various scenarios. The program can give results under fairly wide range of conditions. The forces can be adjusted to be attractive or repulsive, and could be changed further by changing the program. External forces are not included here. Future work will expand this range and look for interesting scenarios to simulate. Two such applications are referenced here. Another application is under way. This application is to subject the e-p particles in the square to an external E field.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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