



Large Deviation Principle for Stochastic Differential System Pertubated by a Rapid Process in the Besov-Orlicz Topology

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This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this study, we want to analyze a large deviation principle associated with a family process $x^\epsilon = (x_t^\epsilon), t \geq 0$ in the Besov-Orlicz space whose drift coefficient is additionally perturbed by a strictly stationary process $\zeta = (\zeta_t), t \geq 0$, defined on the probability space $(\Omega, \mathcal{F}^\zeta, (\mathcal{F}_t^\zeta)_{t \geq 0}, \mathbb{P}_t^\zeta)$, with values in a general Polish space E . The process x^ϵ is a solution of Itô integral:

$$\begin{cases} x_t^\epsilon &= x_0 + \int_0^t b(x_s^\epsilon, \zeta_{s/\epsilon}) ds + \sqrt{\epsilon} \int_0^t \sigma(x_s^\epsilon) dw_s \\ x_0 &= x \in \mathbb{R}^d \end{cases}$$

in which the condition ζ is independent of the brownian motion w and obeys a large deviation principle with a rate function I_ζ .

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1 Introduction

In this paper, we consider a diffusion processes $x^\epsilon = (x_t^\epsilon), t \geq 0$ d-dimensional solution of stochastic differential equation (SDE):

$$x_t^\epsilon = x_0 + \int_0^t b(x_s^\epsilon, \zeta_{s/\epsilon}) ds + \sqrt{\epsilon} \int_0^t \sigma(x_s^\epsilon) dw_s, \quad x_0 = x \in \mathbb{R}^d \quad (1)$$

where w is a Wiener's standard process independent of ζ , where $\zeta = (\zeta_t), t \geq 0$ is a process with values in a general state space E and obeys a large deviation principle with a rate function I_ζ . Our purpose here is to establish the asymptotic evaluation of $P(x^\epsilon \in A)$ where A is a Borel set of Besov-Orlicz space under the assumption that the process x_t^ϵ converges to the solution \bar{x}_t defined by:

$$\begin{aligned} d\bar{x}_t &= \bar{b}(\bar{x}_t) dt, \quad \bar{x}_0 = 0, \\ \bar{b}(x) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T b(x_s, \zeta_s) ds \end{aligned}$$

The asymptotic evaluation obtained will be the result of a large deviation from x_t^ϵ compared to \bar{x}_t .

The basic work on the subject is the article by Freidlin [1], se also refered to Ventcel's book - Freidlin [2] where they gets this evaluation under the assumption:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \log E(\exp(\int_0^T \langle \alpha, b(x_s, \zeta_{s/\epsilon}) \rangle ds)) = H^0(x, \alpha) \quad (2)$$

exists uniformly in x and differentiable in α .

The special case $\zeta \equiv 0$ ($b(x_t^\epsilon, 0) = b(x_t^\epsilon)$) and $\sigma \neq Id$ was studied by Freidlin & Wentzell [3] se also refered to Varadhan [4], Azencott [5] and Stroock [6] with the usual topology uniform, Ben Arous and Ledoux [7] have developed a large deviation principle(LDP) in Hölder's space. Later on, an extension to Besov's space was considered in Eddahbi et al [8] and Roynette's [9]. The particular case $\sigma \equiv 0, \zeta \neq 0$ and $b \neq Id$ have been studied by M. Brancovan [10].

The aim of this paper is to study the large deviation principle (LDP) of the law of $\{x_t^\epsilon, \epsilon > 0\}$ in the Besov-Orlicz topology. This is the extension of the result of H. Lapeyre [11] in a stronger topology.

The paper is organized as follows: In section 2, we introduce some hypotheses and notations. Section 3 contains some preliminary definitions and general results which are essential for the proof of the theorem (4.4). Section 4, under the hypotheses in section 2, we prove in theorem (4.1) the LDP of x_t^ϵ , solution of (1) when ζ satisfies a large deviation principle.

2 Hypotheses and Notations

2.1 Hypotheses

In this paper, we assume that the following hypotheses will be verified:

- H1.** The function $b : \mathbb{R}^d \times E \rightarrow \mathbb{R}^d$ is measurable and satisfying two conditions: linear growth in x and a Lipschitz condition in x and y . In other words, for some constant $C, 0 < C < \infty$

and for all $x, x' \in \mathbb{R}^d$ and $y, y' \in E$ we have:

$$\begin{aligned} \|b(x, y) - b(x', y')\|_{\mathbb{R}^d} &\leq C(\|y - y'\|_{\mathbb{R}^d} + \|x - x'\|_{\mathbb{R}^d}) \\ \|b(x, y)\|_{\mathbb{R}^d} &\leq C(1 + \|x\|_{\mathbb{R}^d}) \end{aligned}$$

H2. The function $\sigma : \mathbb{R}^d \rightarrow \mathcal{M}_{\mathbb{R}^d \times \mathbb{R}^d}$ (the space of $d \times d$ matrices endowed with Hilbert Schmidt norm) is measurable and satisfying a Lipschitz condition in x . In other words, for some constant C , $0 < C < \infty$ and for all $x, x' \in \mathbb{R}^d$ we have:

$$\begin{aligned} \|\sigma(x) - \sigma(x')\|_{\mathbb{R}^d} &\leq C(\|x - x'\|_{\mathbb{R}^d}) \\ \|\sigma(x)\|_{\mathbb{R}^d} &\leq C \end{aligned}$$

H3. W is a standard \mathbb{R}^d -valued Brownian motion

H4. $\zeta = (\zeta_t), t \geq 0$ is a process with values in a general state space E , jointly defined on some stochastic basis $(\Omega, \mathcal{F}_t^\zeta, (\mathcal{F}_t^\zeta)_{t \geq 0}, \mathbb{P}_t^\zeta)$, independent of brownian motion w and obeys a large deviation principle with a good rate function I_ζ .

2.2 Notations

2.2.1 Cameron-Martin space

Let $H(\mathbb{R}^d)$ be the Cameron-Martin space associated with the Brownian motion on \mathbb{R}^d

$$H(\mathbb{R}^d) = \left\{ \begin{array}{l} f : [0, 1] \rightarrow \mathbb{R}^d, f \text{ is absolutely continuous such that} \\ f(0) = 0 \text{ and } \int_0^1 |\dot{f}_s|^2 ds < +\infty \end{array} \right\}$$

$H(\mathbb{R}^d)$ is a Hilbert Space equipped with the scalar product

$$\langle f, g \rangle = \int_0^1 \dot{f}_s \dot{g}_s ds$$

2.2.2 Besov-Orlicz space

Let B_{M_β, w_α} be denote the Besov-Orlicz space of continuous function $f : [0, 1] \rightarrow \mathbb{R}^d$ such that $\|f\|_{M_\beta, w_\alpha} < \infty$. For all $\alpha > 0$, let us put

$$\|f\|_{M_\beta, w_\alpha} = \|f\|_{M_\beta} + \sup_{0 \leq t \leq 1} \frac{w_{M_\beta}(f, t)}{w_{\alpha, \lambda}(t)}$$

where $w_{\alpha, \lambda}(t) = t^\alpha (1 + \log \frac{1}{t})^\lambda, \forall \alpha > 0, \|f\|_{M_\beta} = \inf \left\{ \tau > 0, \frac{1}{\tau} \left[1 + \int_0^1 M_\beta(\tau |f(t)|) dt \right] \right\}$ et $w_{M_\beta}(f, t) = \sup_{0 \leq h \leq t} \|\Delta_h f\|_{M_\beta}$ with

$$\Delta_h f(x) = 1_{[0, 1-h]}(x)(f(x+h) - f(x)), \forall h \in [0, 1].$$

We will use the equivalent of Cieleski, Z. [9]. Let $\chi_1, \chi_{j,k}, j = 0, 1, \dots, k = 1 \dots 2^j, \text{supp} \chi_{j,k} = [(k-1)/2^j, k/2^j]$, be the set of Haar functions over the interval $[0, 1]$, and let $\varphi_0(t) = 1, \varphi_1(t) = t, \varphi_{j,k}(t) = \int_0^t \chi_{j,k}(s) ds$ be the set of Schauder functions. Let $f : [0, 1] \rightarrow \mathbb{R}^d$ be a continuous functions, let us note by $\{A_n(f), n \geq 0\}$ the coefficients of the decomposition of f in the Schauder basis given by

$$f(t) = A_0(f)\varphi_0(t) + A_1(f)\varphi_1(t) + \sum_{n=2^{j+1}}^{2^{j+1}} \sum_{j,k} A_n(f)\varphi_{j,k}(t)$$

where $A_0(f) = f(0)$, $A_1(f) = f(1) - f(0)$ and

$$A_n(f) = 2^{\frac{j}{2}} \left[\left(f\left(\frac{2k-1}{2^{j+1}}\right) - f\left(\frac{2k-2}{2^{j+1}}\right) \right) - \left(f\left(\frac{2k}{2^{j+1}}\right) - f\left(\frac{2k-1}{2^{j+1}}\right) \right) \right]$$

Let B_{M_β, w_α}^0 be the subspace of B_{M_β, w_α} corresponding to the sequence $f_{j,k}$ such that

$$B_{M_\beta, w_\alpha}^0 = \left\{ f \in \mathcal{C}([0, 1], \mathbb{R}^d); \|f\|_{M_2, w_\alpha} < \infty, \lim_{j \wedge p \rightarrow \infty} 2^{-j(\frac{1}{2} - \alpha + \frac{1}{p})} p^{-\gamma} (1+j)^{-\lambda} \|f_{j,\cdot}\|_p = 0 \right\}$$

where

$$\|f_{j,\cdot}\|_p = \left(\sum_{k=1}^{2^j} |f_{j,k}|^p \right)^{\frac{1}{p}} \text{ and } \beta\gamma = 1$$

B_{M_β, w_α}^0 is a Banach space.

For more details on Besov-Orlicz space we refer to instance [9].

3 Preliminary Definitions and Results

3.1 Preliminary definitions

Definition 3.1. A rate function is a function $I : \Xi \rightarrow [0; +\infty]$ on a Hausdorff topological space Ξ which is lower semi-continuous, ie. where all the level set $\Gamma_a = \{x \in \Xi, I(x) \leq a\}$ are closed in Ξ . A rate function $I : \Xi \rightarrow [0; +\infty]$ is called a good rate function, if all the level set $\{x \in \Xi, I(x) \leq a\}$ for $a \geq 0$ are compact in Ξ

Definition 3.2. A family $\{P^\varepsilon\}_{\varepsilon>0}$ of probabilities measures on Hausdorff topological space Ξ satisfies the large deviation principle (or shorter LDP) with rate function $I : \Xi \rightarrow [0; +\infty]$, if the following two estimates hold:

i) (Lower bound.) For every open subset \mathcal{O} of Ξ

$$\liminf_{\varepsilon \rightarrow 0} \varepsilon \log P_\varepsilon(\mathcal{O}) \geq -I(\mathcal{O})$$

ii) (Upper bound.) For every closed subset \mathcal{F} of Ξ

$$\limsup_{\varepsilon \rightarrow 0} \varepsilon \log P_\varepsilon(\mathcal{F}) \leq -I(\mathcal{F})$$

3.2 Preliminary results

We will use the following characterization theorem.

Theorem 3.3. Let $p_0 \geq 1$, f belongs to B_{M_β, w_α}^0 if and only if

$$\max \left(|f_0|, |f_1|, \sup_{p \geq p_0} \sup_{j \geq 0} 2^{-j(\frac{1}{2} - \alpha + \frac{1}{p})} p^{-\gamma} (1+j)^{-\lambda} \|f_{j,\cdot}\|_p \right) < \infty \tag{3}$$

Theorem 3.4. Let f belongs to B_{M_β, w_α}^0 if and only if

$$\lim_{j \vee p \rightarrow p_0} 2^{-j(\frac{1}{2} - \alpha + \frac{1}{p})} p^{-\gamma} (1+j)^{-\lambda} \|f_{j,\cdot}\|_p < \infty \tag{4}$$

For the proof of this result we refer to [9]

Consider the following norm which is are crucial to prove our results:

$$\|f\|_{**} = \sup_{0 \leq s < t \leq 1} \frac{|f(t) - f(s)|}{w(t-s)}$$

this is dominated by

$$\|f\|_{**} = \max \left(|f(1)|, \sup_{j \geq 0} \sup_{0 \leq k \leq 2^j} \frac{|f_{j,k}|}{\sqrt{1+j}} \right).$$

It is easy to show that there exist $D_1 > 0$ and $D_2 > 0$ such that

$$\|f\| \leq D_1 \|\|f\|_{M_2, w} \leq D_1 \|f\|_{**} \leq D_2 \|f\|_*$$

The following LDP proved by Baldi et al. (1992) extends the classical Schilder theorem (see Schilder 1996; Deuschel and Strook 1989)

Theorem 3.5. *Let P^ε be the law of $\sqrt{\varepsilon}w$ on $\mathcal{B}_{M_2, w_\alpha}^0$ equipped with the norm $\|\cdot\|_{M_\beta, w_\alpha}$ satisfying the LDP with the good rate function $S^W(\cdot)$ defined by:*

$$S^W(h) = \begin{cases} \frac{1}{2} \int_0^T |\dot{h}(s)|^2 ds & \text{if } h \in H(\mathbb{R}^d) \\ +\infty & \text{otherwise} \end{cases}.$$

One of the basic tools in large deviation theory is the 'contraction principle' (see Deuschel and Strook 1989). It enables the new rate function to be computed after the data have been transformed by a continuous map [12].

Theorem 3.6. *Let Q^ε be a family of probability measure on a Polish space E and satisfies the LDP with a good rate function λ .*

Let $F : E \rightarrow E'$ be continuous. Denote by $Q^\varepsilon = P^\varepsilon \circ F^{-1}$ the family of image measure of P^ε , then $\{Q^\varepsilon\}$ satisfies the LDP with a good rate function $\tilde{\lambda}$ defined by

$$\tilde{\lambda}(y) = \inf_{x: F(x)=y} \lambda(x).$$

Lemma 3.7. *There exist $C = C_l$ such that for all $\lambda > 0$ and $\mu > 0$ where $\lambda > 4l\mu > 0$ and $\lambda > 2\sqrt{\log 2}$, we have*

$$P \left[\|W\|_{**} \geq \lambda, \|W\| \leq \mu \right] \leq C \max \left(1, l \left(\frac{\lambda}{4l\mu} \right)^2 \exp \left(- \frac{\lambda^2}{C} \ln \left(\frac{\lambda}{4l\mu} \right) \right) \right) \quad (5)$$

Lemma 3.8. *(Exponential inequality)*

For all $u > 2\sqrt{\log 2}$ and for all process K on $[0, 1]$ there exist $C = C_l$ such that

$$P \left[\left\| \int_0^\cdot K_s dw_s \right\|_{**} \geq u, \|K\| \leq 1 \right] \leq C \exp \left(- \frac{u^2}{C} \right). \quad (6)$$

Now, we give a new formulation for the contraction principle which will be needed later.

Lemma 3.9. *Let $(E_x, d_x), (E_y, d_y), (E_z, d_z), (E, d)$ denote Polish spaces and $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space.*

Suppose that $(x^\varepsilon, \varepsilon > 0)$ is a family of random variables with values by satisfying a LDP with a rate function I_x , and $(y^\varepsilon, \varepsilon > 0)$ a random variable with values by satisfying a LDP with a rate function

I_y .

Suppose that for each $\varepsilon > 0$, x^ε is independent of y^ε then the family of random variable $z = F(x^\varepsilon, y^\varepsilon)$ by satisfying a LDP with rate function $I_F(z)$ defined by

$$I_F(z) = \inf_{F(x,y)=z} \left\{ I_x(x) + I_y(y) \right\}$$

where $F : E_x \times E_y \rightarrow E_z$ is continuous.

The main purpose of the following section is to build a functional controlling the large deviation of x^ε on $\mathcal{B}_{M_2, w_\alpha}^0$ if we know the large deviation of ζ in E . More precisely, we are building for all $T > 0$ a functional S_T satisfying the following assertions:

- i) For each positive α , $K_\alpha = \left\{ \varphi \in \mathcal{B}_{M_2, w_\alpha}^0 / S_T(\varphi) \leq \alpha \right\}$ is a compact set
- ii) For every open subset O of $\mathcal{B}_{M_2, w_\alpha}^0$,

$$\overline{\lim}_{\varepsilon \rightarrow 0} \varepsilon \log P(x^\varepsilon(x) \in O) \geq - \inf_{\varphi \in O} S_T(\varphi).$$

- iii) For every closed F of $\mathcal{B}_{M_2, w_\alpha}^0$,

$$\overline{\lim}_{\varepsilon \rightarrow 0} \varepsilon \log P(x^\varepsilon(x) \in F) \leq - \inf_{\varphi \in F} S_T(\varphi).$$

The aim of this study is to establish the large deviation principle of x^ε in $\mathcal{B}_{M_2, w_\alpha}^0$ by using the Azencott's method in a general setting. As a reminder for Azencott's method, let (E_i, d_i) , $i = 1, 2$ be two Polish spaces and $x_\varepsilon^i \rightarrow E_i$, $\varepsilon > 0$, $i = 1, 2$ two families of random variables. Assume that $\{x_1^\varepsilon, \varepsilon > 0\}$ satisfies a LDP with rate function $I_1 : E_1 \rightarrow [0, +\infty]$. Let $\Phi : \{I_1 < \infty\} \rightarrow E_2$ be a mapping such that its restriction to the compact sets $\{I_1 \leq a\}$ is continuous in the topology of E_1 . For any $g \in E_2$ we set $I(g) = \text{Inf} \left\{ I_1(f), \Phi(f) = g \right\}$. Suppose that for $R, \rho, a > 0$ there exist α and $\varepsilon_0 > 0$ such that for $f \in E_1$ satisfying $I_1(f) \leq a$ and $\varepsilon \leq \varepsilon_0$ we have

$$P \left\{ d_2(x_2^\varepsilon, \Phi(f)) \geq \rho, d_1(x_1^\varepsilon, f) \leq \alpha \right\} \leq \exp\left(-\frac{R}{\varepsilon^2}\right) \tag{7}$$

Then the family $\{x_2^\varepsilon, \varepsilon > 0\}$ satisfies a LDP with rate function I .

4 The Main Result

Let $f, g : [0, T] \rightarrow \mathbb{R}^d$ be absolutely continuous, denote by $B_x(g, f)$ the unique solution of ordinary differential equation:

$$\dot{\varphi}_t = \dot{g}_t + \sigma(\varphi_t) \dot{f}_t \tag{8}$$

Let E be a locally countable space and \mathcal{E} is the Borel σ -field. Let us denote by $\mathcal{M}_1(E)$ a set of probability measures on (E, \mathcal{E}) , equipped with the topology of weak convergence.

Now, let us introduce the mapping $\lambda^\zeta : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \overline{\mathbb{R}}_+$ defined by:

$$\lambda^\zeta(g, f) = \text{Inf} \left\{ I_\zeta(\mu), \mu \in \mathcal{M}_1(E), \int_E b(g, y) \mu(dy) = f \right\} \tag{9}$$

We denote for any (Φ, Ψ) absolutely continuous from $[0, T]$ to \mathbb{R}^d ,

$$\kappa^\zeta(\Phi, \Psi) = \begin{cases} \int_0^T \lambda^\zeta(\Phi_s, \dot{\Psi}_s) ds & \text{if } \Psi \text{ is absolutely continuous} \\ +\infty & \text{otherwise} \end{cases}$$

It is a remarkable fact, the function κ^ζ is lower semi-continuous from $\mathbb{R}^d \times \mathbb{R}^d$ to $\overline{\mathbb{R}}_+$, convex to second argument and denote by $\kappa^\zeta(\Phi)$ the mapping defined by $\kappa^\zeta(\Phi) = \kappa^\zeta(\Phi, \Phi)$ for any function Φ absolutely continuous

Theorem 4.1. Assume that H_1, H_2 . Let x^ε is the unique solution of (1).

Then the family $\{x^\varepsilon\}_{\varepsilon>0}$ satisfies a LDP in $\mathcal{B}_{M_2, w_\alpha}^0$ with a good rate function $S_T(\varphi)$ defined by

$$S_T(\varphi) = \inf_{(g, f)} \left\{ S^W(f) + \kappa^\zeta(g), \varphi = B_x(g, f) \right\}$$

where $B_x(g, f)$ is defined in (8) and S^W is defined in **Theorem 3.5**

For the proof of the Theorem (4.1), we will be interested in the behavior of x^ε in a tube around a function φ absolutely continuous in $\mathcal{B}_{M_2, w_\alpha}^0$. In this kind of tube, we compare x^ε to x_φ^ε solution of $dx_\varphi^\varepsilon = b(x_\varphi^\varepsilon, \xi_{t/\varepsilon}) dt + \sqrt{\varepsilon} \sigma(x_\varphi^\varepsilon) dw_t$, in other words, we will show that for all $\delta > 0$, for all continuous function φ , there exist $\delta_1 > 0$ such that $P\left(\|x^\varepsilon - \varphi\|_{\mathcal{B}_{M_2, w_\alpha}^0} \leq \delta\right) \leq P\left(\|x_\varphi^\varepsilon - \varphi\|_{\mathcal{B}_{M_2, w_\alpha}^0} \leq \delta_1\right)$.

It is easy to check it by using the exponential inequality. For absolutely continuous functions $\varphi \in \left(\mathcal{C}([0, T], \mathbb{R}^d)\right)$, the mapping $F^\varphi : \left(\mathcal{C}([0, T], \mathbb{R}^d)\right) \times \mathcal{B}_{M_2, w_\alpha}^0 \rightarrow \mathcal{B}_{M_2, w_\alpha}^0$ defined by

$$F^\varphi(g, f) = h \text{ if and only if } h_t = x + g_t + \sigma(\varphi_t) f_t + \int_0^t f_s d\sigma(\varphi_s)$$

is continuous and $x^{\varepsilon, \varphi}$ is the image of $(y^{\varepsilon, \varphi}, \sqrt{\varepsilon}w)$ by F^φ

$$\text{where } dy_t^{\varepsilon, \varphi} = b(y_t^{\varepsilon, \varphi}, \xi_{t/\varepsilon}) dt, \quad y_0^{\varepsilon, \varphi} = 0. \tag{10}$$

Let $L^0(x, \alpha)$ be the conjugate of the quadratic convex function $H^0(x, \alpha)$ obtained from the formula in (2). L^0 is lower semicontinuous(lsc), with values in $\mathbb{R}_+ \cup \{\infty\}$, convex to second argument

For some couple values (φ, ψ) in $\mathcal{B}([0, T], \mathbb{R}^d)$, denoted by:

$$\begin{cases} S^0(\varphi, \psi) &= \int_0^T L^0(\varphi_s, \dot{\psi}_s) ds \text{ if } \psi \text{ is absolutely continuous} \\ &= +\infty \text{ otherwise} \\ S^W(\psi) &= \int_0^T \frac{1}{2} |\dot{\psi}_s|^2 ds \text{ if } \psi \text{ is absolutely continuous} \\ &= +\infty \text{ otherwise} \end{cases}$$

Proposition 4.2. For absolutely continuous functions $\varphi \in \left(\mathcal{C}([0, T], \mathbb{R}^d)\right)$ then $S^0(\varphi, \cdot)$ is a rate function of $y^{\varepsilon, \varphi}(0)$ in $\left(\mathcal{C}([0, T], \mathbb{R}^d)\right)$ (see [1]).

Proposition 4.3. Assume (H_4) , the couple of random variables $(y^{\varepsilon, \varphi}(0), \sqrt{\varepsilon}w)$ considered a random variable with values in $\mathcal{B}_{M_2, w_\alpha}^0$ satisfying LDP with the following rate function $S^\varphi(g, f)$ defined by:

$$S^\varphi(g, f) = S^0(\varphi, g) + S^W(f) \tag{11}$$

By using the contraction principle, the law of x^ε satisfies LDP on $\mathcal{B}_{M_2, w_\alpha}^0$ with the rate function defined by:

$$S\varphi(\omega) = \inf \left\{ S^\varphi(g, f), \omega = F^\varphi(g, f) \right\}. \tag{12}$$

Now we aim to establish in Theorem (4.1) a large deviation principle (LDP) for the family x^ε on the Besov-Orlicz Space $\mathcal{B}_{M_2, w_\alpha}^0$ by using the Azencott's method mentioned above to the random variables $x_1^\varepsilon = \sqrt{\varepsilon}w$ and $x_2^\varepsilon = x_\varphi^\varepsilon$

Theorem 4.4. For any $r, \alpha, a > 0$, for each x with values in \mathbb{R}^d , there exist $\rho, \tilde{r}, \varepsilon_0$ depending only on r, α, a, x such that if g, f absolutely continuous verifying $\| \dot{f} \| \leq a$ and $\varphi = B_y(g, f)$, $|x - y| \leq \tilde{r}$, $\varepsilon \leq \varepsilon_0$ we have,

$$\mathbb{P} \left(\| x_\varphi^\varepsilon - \varphi \|_{**} > \alpha, \| y^{\varepsilon, \varphi}(0) - g \|_{C_0(\mathbb{R}^d)} < \rho, \| \sqrt{\varepsilon} - f \|_{C_0(\mathbb{R}^d)} < \rho \right) \leq \exp\left(-\frac{r}{\varepsilon}\right).$$

where $\varphi = B_x(g, f)$ if and only if $\dot{\varphi}_t = \dot{g}_t + \sigma(\varphi)\dot{f}_t, \varphi_0 = 0$

Proof of Theorem 4.4. Indeed, let $w^f = w - \frac{1}{\sqrt{\varepsilon}}f$. Girsanov's theorem implies that w^f is a d-dimensional Wiener process with respect to the probability P^f given by

$$\frac{dP^f}{dP} = \exp \left(\frac{1}{\sqrt{\varepsilon}} \int_0^1 \dot{f}_s dw_s - \frac{1}{\varepsilon} \int_0^1 |\dot{f}_s|^2 ds \right)$$

Let $\{Y_t^\varepsilon, 0 \leq t \leq 1\}$ be the solution of SDE

$$Y_t^\varepsilon = x + \int_0^t \left(b(Y_s^\varepsilon, \zeta_{s/\varepsilon}) + \sigma(Y_s^\varepsilon)\dot{f}_s \right) ds + \sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f, P^f \text{ almost surely} \quad (13)$$

To simplify the notation, set for any $\rho, \alpha, \varepsilon > 0$

$$U^f = \left\{ \| Y^\varepsilon(x) - \varphi \|_{M_2, w_\alpha} > \alpha, \| y^{\varepsilon, \Phi}(0) - g \|_{C_0(\mathbb{R}^d)} < \rho, \| \sqrt{\varepsilon}w - f \|_{C_0(\mathbb{R}^d)} < \rho \right\}$$

And

$$V^f = \exp \left\{ \left| \frac{1}{\sqrt{\varepsilon}} \int_0^1 \dot{f}_s dw_s \right| > \frac{\lambda}{\sqrt{\varepsilon}} \right\}.$$

Then

$$\begin{aligned} P(U^f) &\leq P \left\{ U^f \cap \left(V^f \leq \exp \left(\frac{\lambda}{\varepsilon} \right) \right) \right\} + P \left\{ V^f > \frac{\lambda}{\varepsilon} \right\} \\ &\leq \exp \left(\frac{\lambda + a/2}{\varepsilon} \right) P^f(U^f) + P \left(\left| \frac{1}{\sqrt{\varepsilon}} \int_0^1 \dot{f}_s dw_s \right| \geq \frac{\lambda}{\varepsilon} \right) \end{aligned} \quad (14)$$

where $a = \| h \|_H^2$ and $\lambda \in \mathbb{R}$

By the classical exponential inequality,

$$P \left(\left| \int_0^1 \dot{f}_s dw_s \right| \geq \frac{\lambda}{\sqrt{\varepsilon}} \right) \leq 2 \exp \left(-\frac{\lambda^2}{2a\varepsilon} \right) \leq \exp \left(-\frac{r}{\varepsilon} \right). \quad (15)$$

Set

$$Y^\varepsilon(w^f) = x^\varepsilon(w^f + \frac{1}{\sqrt{\varepsilon}}f).$$

Consequently, we obtain :

$$P^f(U^f) = P \left(\| Y^\varepsilon(x) - \varphi \|_{**} > \alpha, \| y^{\varepsilon, \Phi}(0) - g \|_{C_0(\mathbb{R}^d)} < \rho, \| \sqrt{\varepsilon}w \|_{C_0(\mathbb{R}^d)} < \rho \right),$$

where Y^ε is the solution of SDE in (13), the estimate (14) and (15) complete the proof of the theorem (4.4).

The aim of proof of theorem 4.4 is an immediate consequence of the next following propositions.

For any $n \in \mathbb{N}^*$ we consider the approximation sequence of the process Y^ε defined by

$$Y_t^{\varepsilon,n} = Y_{\frac{j}{2^n}}^\varepsilon, \text{ if } s \in [\frac{j}{2^n}, \frac{j+1}{2^n}[\text{ for all } j = 0, 1, 2, \dots, 2^n - 1$$

Proposition 4.5. For all $r > 0$ and $\gamma > 0$ there exist $\varepsilon_0 > 0$ and n such that if $0 < \varepsilon < \varepsilon_0$, we have:

$$P^f \left\{ \| Y^\varepsilon - Y^{\varepsilon,n} \|_{C_0(\mathbb{R}^d)} \geq \gamma \right\} \leq \exp(-\frac{r}{\varepsilon})$$

Proof of Proposition 4.5. For a detailed proof of Proposition 4.5, we refer to Priouret,P(1982, Lemma 2) [13]

Proposition 4.6. For every $\gamma_1 > 0$, $\rho > 0$ then

$$P^f(U^f) \leq P^f \left(\| \sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f \|_{**} > \gamma_1, \| \sqrt{\varepsilon} w^f \|_{C_0(\mathbb{R}^d)} < \rho \right).$$

Proof of Proposition (4.6).

$$\begin{aligned} Y_t^\varepsilon - \varphi &= x - y + \int_0^t [b(Y_s^\varepsilon, \xi_{s/\varepsilon}) + \sigma(Y_s^\varepsilon) \dot{f}_s] ds + \sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f \\ &\quad - \int_0^t [b(\varphi_s, \xi_{s/\varepsilon}) + \sigma(\varphi_s) \dot{f}_s] ds + y_t^{\varepsilon,\varphi} - g_t \\ &= x - y + \int_0^t [b(Y_s^\varepsilon, \xi_{s/\varepsilon}) - b(\varphi_s, \xi_{s/\varepsilon})] ds + \int_0^t [\sigma(Y_s^\varepsilon) - \sigma(\varphi_s)] \dot{f}_s ds \\ &\quad + \sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f + y_t^{\varepsilon,\varphi} - g_t \\ &= x - y + \int_0^t \Phi(\xi_{s/\varepsilon}, Y_s^\varepsilon, \varphi_s) ds + \int_0^t \Psi(Y_s^\varepsilon, \varphi_s) ds + \sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f + y_t^{\varepsilon,\varphi} - g_t \end{aligned}$$

where

$$\Phi(z, x, y) = b(x, z) - b(y, z)$$

$$\Psi(x, y) = (\sigma(x) - \sigma(y)) \dot{f}$$

Let us first give an estimate of :

$$\| \int_0^t \Phi(\xi_{s/\varepsilon}, Y_s^\varepsilon, \varphi_s) ds + \int_0^t \Psi(Y_s^\varepsilon, \varphi_s) ds \|_{**} \tag{16}$$

From the definition of $\| \cdot \|_{**}$ we have

$$\| \int_0^t \Phi(\xi_{s/\varepsilon}, Y_s^\varepsilon, \varphi_s) ds + \int_0^t \Psi(Y_s^\varepsilon, \varphi_s) ds \|_{**} \leq M_1 + M_2$$

where

$$M_1 \leq \sup_{0 \leq u < v \leq 1} \frac{\int_u^v \| \Phi(\xi_{s/\varepsilon}, Y_s^\varepsilon, \varphi_s) \|_{\mathbb{R}^d} ds}{w(u-v)} + \sup_{0 \leq u < v \leq 1} \frac{\int_0^u \| \Phi(\xi_{s/\varepsilon}, Y_s^\varepsilon, \varphi_s) \|_{\mathbb{R}^d} ds}{w(u-v)}$$

and

$$M_2 \leq \sup_{0 \leq u < v \leq 1} \frac{\int_u^v \| \Psi(Y_s^\varepsilon, \varphi_s) \|_{\mathbb{R}^d} ds}{w(u-v)} + \sup_{0 \leq u < v \leq 1} \frac{\int_0^u \| \Psi(Y_s^\varepsilon, \varphi_s) \|_{\mathbb{R}^d} ds}{w(u-v)}$$

Therefore, assumption **H1** and **H2** we have

$$\begin{aligned}
 M_1 &\leq \sup_{0 \leq u < v \leq 1} \frac{C \int_u^v \|Y_s^\varepsilon - \varphi_s\|_{\mathbb{R}^d} ds}{w(u-v)} + \sup_{0 \leq u < v \leq 1} \frac{C \int_0^u \|Y_s^\varepsilon - \varphi_s\|_{\mathbb{R}^d} ds}{w(u-v)} \\
 M_1 &\leq C K \|Y_s^\varepsilon - \varphi_s\|_{**} \sup_{0 \leq u < v \leq 1} \frac{|u| + |v-u|}{w(u-v)} \\
 M_2 &\leq \sup_{0 \leq u < v \leq 1} \frac{\int_u^v \|\Psi(Y_s^\varepsilon, \varphi_s)\|_{\mathbb{R}^d} ds}{w(u-v)} + \sup_{0 \leq u < v \leq 1} \frac{\int_0^u \|\Psi(Y_s^\varepsilon, \varphi_s)\|_{\mathbb{R}^d} ds}{w(u-v)} \\
 M_2 &\leq \sup_{0 \leq u < v \leq 1} \frac{C \int_u^v \|Y_s^\varepsilon - \varphi_s\|_{\mathbb{R}^d} |\dot{f}_s| ds}{w(u-v)} + \sup_{0 \leq u < v \leq 1} \frac{C \int_0^u \|Y_s^\varepsilon - \varphi_s\|_{\mathbb{R}^d} |\dot{f}_s| ds}{w(u-v)} \\
 M_2 &\leq C K \|Y_s^\varepsilon - \varphi_s\|_{**} \|\dot{f}_s\|_{H(\mathbb{R}^d)} \sup_{0 \leq u < v \leq 1} \frac{|u| + |v-u|}{w(u-v)}
 \end{aligned}$$

Consequently

$$M_1 + M_2 \leq C K L \|Y_s^\varepsilon - \varphi_s\|_{**} \left(1 + \|\dot{f}_s\|_{H(\mathbb{R}^d)}\right)$$

where

$$L = \sup_{0 \leq u < v \leq 1} \frac{|u| + |v-u|}{w(u-v)}$$

Thus there exist α and β such that

$$\|Y_s^\varepsilon - \varphi_s\|_{**} \leq \alpha + \beta \|Y_s^\varepsilon - \varphi_s\|_{**} + \|\sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f\|_{**}$$

where

$$\alpha = \tilde{r} + \|y^{\varepsilon, \varphi} - g\|_{C_0(\mathbb{R}^d)}$$

and

$$\beta = C K L \|Y_s^\varepsilon - \varphi_s\|_{**} \left(1 + \|\dot{f}_s\|_{H(\mathbb{R}^d)}\right)$$

It follows that

$$\|Y_s^\varepsilon - \varphi_s\|_{**} \leq \frac{\alpha}{1-\beta} + \frac{1}{1-\beta} \|\int_0^t \sqrt{\varepsilon} \sigma(Y_s^\varepsilon) dw_s^f\|_{**}$$

Recall that if X et Y are two real random variables and if $X(\omega) < Y(\omega)$ for any $\omega \in \Omega$ thus $\{\omega \in \Omega \text{ such that } X(\omega) \leq a\} \subset \{\omega \in \Omega \text{ such that } Y(\omega) \leq a\}$ So we deduced that :

$$P^f(U^f) \leq P^f\left(\|\sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f\|_{**} > \gamma_1, \|\sqrt{\varepsilon} w^f\|_{C_0(\mathbb{R}^d)} < \rho\right).$$

Proposition 4.7. For all $r > 0, \gamma_1 > 0$, there exist $\varepsilon > 0$ and $\rho > 0$ such that

$$P^f\left(\|\sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f\|_{**} > \gamma_1, \|\sqrt{\varepsilon} w^f\|_{C_0(\mathbb{R}^d)} < \rho\right) \leq \exp\left(-\frac{r}{\varepsilon}\right).$$

Proof of Proposition (4.7). For $\alpha > 0$ and for every $n \in \mathbb{N}$, we have

$$A = \left\{ \|\sqrt{\varepsilon} \int_0^t \sigma(Y_s^\varepsilon) dw_s^f\|_{**} \geq \rho, \|\sqrt{\varepsilon} w^f\|_{C_0(\mathbb{R}^d)} \leq \alpha \right\} \subset A_1 \cup A_2 \cup A_3$$

where

$$\begin{cases} A_1 = \left\{ \|\sqrt{\varepsilon} \int_0^\cdot [\sigma(Y_s^\varepsilon) - \sigma(Y_s^{\varepsilon,n})] dw_s^f\|_{**} \geq \frac{\rho}{2}, \|Y^\varepsilon - Y^{\varepsilon,n}\|_{C_0(\mathbb{R}^d)} \leq \gamma \right\} \\ A_2 = \left\{ \|Y^\varepsilon - Y^{\varepsilon,n}\|_{C_0(\mathbb{R}^d)} \geq \gamma \right\} \\ A_3 = \left\{ \|\sqrt{\varepsilon} \int_0^\cdot \sigma(Y_s^{\varepsilon,n}) dw_s^f\|_{**} \geq \frac{\rho}{2}, \|\sqrt{\varepsilon} w^f\|_{C_0(\mathbb{R}^d)} \leq \alpha \right\} \end{cases}$$

By using the Proposition 4.5, we obtain: for all $r > 0$ and $\gamma > 0$ there exist ε_0 and n such that for every $0 < \varepsilon < \varepsilon_0$, we have:

$$P^f(A_2) \leq \exp\left(-\frac{r}{\varepsilon}\right)$$

It is easy to check that if $\|Y^\varepsilon - Y^{\varepsilon,n}\|_{C_0(\mathbb{R}^d)} \leq \gamma$ we get $\|\sqrt{\varepsilon}[\sigma(Y_s^\varepsilon) - \sigma(Y_s^{\varepsilon,n})]\|_{**} \leq 4\varepsilon M^2 \gamma^2$. By using the lemma (3.8),

$$P^f(A_1) \leq C \exp\left(-\frac{\rho^2}{C\gamma^2\varepsilon}\right)$$

It should increase $P^f(A_3)$. So we have

$$\begin{aligned} \|\sqrt{\varepsilon} \int_0^\cdot \sigma(Y_s^{\varepsilon,n}) dw_s^f\|_{**} &= \sqrt{\varepsilon} \left\| \sum_{j=0}^n \sigma(Y_{t_j^{\varepsilon,n}}^{\varepsilon,n}) [w^f(t_{j+1} \wedge \cdot) - w^f(t_j \wedge \cdot)] \right\|_{**} \\ &\leq \sqrt{\varepsilon} \sum_{j=0}^n \|\sigma(Y_{t_j^{\varepsilon,n}}^{\varepsilon,n}) [w^f(t_{j+1} \wedge \cdot) - w^f(t_j \wedge \cdot)]\|_{**} \\ &\leq 2\sqrt{\varepsilon} C n \|w^f\|_* . \end{aligned}$$

because the norm $\|\cdot\|_{**}$ is dominated by $\|\cdot\|_*$

By using the lemma (3.7), we have:

$$P^f(A_3) \leq C \max\left(1, \left(\frac{\rho}{16lMn\alpha}\right)^2\right) \exp\left(-\frac{\rho^2}{C\varepsilon 16M^2n^2} \log\left(\frac{\rho}{16lMn\alpha}\right)\right)$$

where C is a constant depending on l et M .

Let $r > 0$ et $\rho > 0$, we choose then $\gamma > 0$ small enough that $\frac{\rho}{C\gamma^2} > r$, and n such that

$$P^f(A_1) \leq C \exp\left(-\frac{r}{\varepsilon}\right)$$

and finally $\left(\frac{\rho^2}{16M^2n^2} \log\left(\frac{\rho}{16lMn\alpha}\right)\right) > Cr$ in (17). This ends the proof of the proposition.

4.1 Construction of the rate function

For any $(x, \alpha) \in (\mathbb{R}^d)^2$, denote by $H(x, \alpha) = H^0(x, \alpha) + 12 \langle \alpha, \Sigma_x \alpha \rangle$ the quadratic function associated to $\sigma(x)$ so $\Sigma_x = \sigma(x)^t \sigma(x)$.

Let us suppose that $L(x, \beta)$ the conjugate quadratic function of $H(x, \alpha)$. L is lower semicontinuous function with values $\mathbb{R}_+ \cup \{+\infty\}$, converged to β , verified by the following: for all $\psi \in \mathcal{B}([0, T], \mathbb{R}^d)$, we denote by

$$S(\varphi, \psi) = \begin{cases} \int_0^T L(s, \psi_s) ds, & \text{if } \psi \text{ is an absolutely continuous,} \\ +\infty & \text{otherwise.} \end{cases} \quad (17)$$

Theorem 4.8. For the absolutely continuous \mathbb{R}^d – value function ψ , let be $S(\cdot, \psi)$ the formula defined in (17) and S^{ψ} the rate function defined in (12). Then there exist a couple of absolutely continuous functions (g, f) verified by $\psi = F(g, f)$ and we obtain $S(\cdot, \psi)$ and S^{ψ} coincide.

Proof of theorem 4.8. We denote for $(x, \alpha) \in (\mathbb{R}^d)^2$, $H(x, \alpha) = H^0(x, \alpha) + 12 \langle \alpha, \Sigma_x \alpha \rangle$. Q_x denotes the quadratic form on \mathbb{R}^d associated with the matrix $\sigma(x)$, defined by

$$Q_x(v) = \langle v, (x)(x)^*v \rangle = \inf \left\{ |w|^2, (x)w = v, v \in^d \right\}.$$

We denote for $(x, \beta) \in (\mathbb{R}^d)^2$,

$$L(x, \beta) = \inf \left\{ L^0(x, \gamma) + Q^*(\cdot); b(\gamma) = \beta \right\}$$

where Q^* is the quadratic form Q_x .

Let $\tau = B_x(g, f)$ be the solution of $\dot{\tau}_t = b(\dot{g}_t) + (\tau_t)\dot{f}_t$

$$\begin{aligned} S^0(g) + S^W(f) &= \int_0^T L^0(s, \dot{g}_s) + 12|\dot{f}_s|^2 ds \\ &\geq \int_0^T L^0(s, \dot{g}_s) + 12 \inf \left\{ |\dot{g}_s|^2; (\tau_s)\dot{g}_s \nabla_s \right\} ds \\ &\geq \int_0^T L^0(s, \dot{g}_s) + 12Q_s^*(\nabla_s) ds \\ &\geq \int_0^T \inf \left\{ L^0(s, \dot{g}_s) + 12Q_s^*(\nabla_s); b(\dot{g}_s) + \nabla_s = \dot{\tau}_s \right\} ds \\ &\geq \int_0^T L^0(s, \dot{\tau}_s) ds \end{aligned}$$

So,

$$S(\tau_s) \geq S(\tau).$$

To check the other inequality, consider $A_x[v]$ defined by

$$A_x[v] = \left\{ w \text{ tel que } (x)w = v, v \in^d \right\}$$

Consider the Borel set Γ defined by

$$\Gamma = \left\{ (x, v) \in U \times^d \text{ such that } A_x[v] \text{ is not empty} \right\}$$

For each $(x, v) \in \Gamma$, we put

$$K(x, v) = \left\{ w \in^d \text{ such that } |w| = \inf |u|; u \in A_x[v] \right\}$$

The mapping $K : \Gamma \rightarrow \{\text{compact in } \mathbb{R}^d\}$ is a measurable family of non-empty compact toward so Rockafeller [14]. Subsequently, there exist a Borelian function $\chi : \Gamma \rightarrow^d$ such that $\chi(x, v) \in K(x, v)$ for $(x, v) \in \Gamma$.

For each ψ such that $S(\cdot, \psi) < +\infty$ we put Ω as set of (x, β) such that $L(x, \beta) < +\infty$ such as

$$S(\cdot, \psi) = \int_0^T L(s, \psi_s) ds$$

As

$$Q_x(v) = \langle v, (x)(x)^* v \rangle = \| (x)v \|^2$$

and

$$Q_x^*(v) = \inf \left\{ |w|^2, w \in A_x[v] \right\},$$

we have

$$S(\cdot, \psi) = \int_0^T L(s, \psi_s) ds = \int_0^T \inf \left\{ L^0(s, \dot{g}_s) + 12Q_s^*(\nabla_s); b(\dot{g}_s) + \nabla_s = \dot{\tau}_s \right\} ds.$$

So there exist a functional $f \in C_0(\mathbb{R}^d)$ such that

$$S(\varphi, \psi) \leq \int_0^T \inf \left\{ L^0(\varphi_s, \dot{g}_s) + 12|\dot{f}|^2 \right\} ds.$$

It is fair enough to ask $\dot{f}_s = |\chi(\varphi_s, \nabla_s)|$ for almost everything $s \in [0, T]$.

4.2 Regularity of the solution in the Besov-Orlicz space

It is clear that the process $\int_0^t b(x_s^\epsilon, \zeta_{s/\epsilon}) ds, t \in I$ belongs a.s. to $B_{M_2, w}^{\varphi, 0}$. Then, it remains to show

that the process $\int_0^t \sigma(x_s^\epsilon) dw_s, t \in I$ satisfies (3) and (4). We will prove the result in the case $k = d = 1$. The extension in the general case is easily deduced

Let us put

$$Y_t = \int_0^t \sigma(x_s^\epsilon) dw_s$$

We will show that for some p_0 , we have for any $\alpha < \frac{1}{2}$

$$\sup_{j \geq 0} \sup_{p \geq p_0} \frac{2^{-j/p}}{p^{1/2}(1+j)^\alpha} \left[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right]^{1/p} < \infty \quad p.s. \tag{18}$$

$$\lim_{j \vee p \rightarrow p_0} \frac{2^{-j/p}}{p^{1/2}(1+j)^{1/2}} \left[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right]^{1/p} = 0 \tag{19}$$

To check the relation 18, let $\lambda > 0$. Using Chebychev inequality, we can get

$$P\left(\frac{2^{-j/p}}{p^{1/2}(1+j)^\alpha} \left[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right]^{1/p} > \alpha\right) \leq \frac{\lambda^{-p} 2^{-j}}{\sqrt{\frac{p}{2}}(1+j)^{\alpha p}} \left(\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right)$$

$|A_n(Y)|$ is dominated by the terms of :

$$A := \left| \int_0^t f_{\frac{2k-1}{2^{j+1}}, \frac{2k}{2^{j+1}}}(s) dw_s \right| \quad \text{et} \quad B := \left| \int_0^t f_{\frac{2k-2}{2^{j+1}}, \frac{2k-1}{2^{j+1}}}(s) dw_s \right|,$$

where

$$f_{r,t}(s) = 1_{r < s \leq t} \sigma(t, x_s) + 1_{s < r \leq t} [\sigma(t, x_s) - \sigma(r, x_s)].$$

For integers $p \geq 2$, using the inequality of Barlow-Yor(1982), for A and B , there exist a constant C_p appearing in the Burkholder-Davis-Gundy inequality such that

$$E|A_n(Y)|^p \leq CM^p p^{p/2}.$$

Hence,

$$\begin{aligned} P\left(\frac{2^{-j/p}}{p^{1/2}(1+j)^\alpha} \left[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right]^{1/p} > \alpha\right) &\leq \frac{\lambda^{-p} 2^{-j}}{\sqrt{\frac{p}{2}}(1+j)^{\alpha p}} \left(\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right) \\ &\leq \left(\frac{C}{\lambda}\right)^p \frac{1}{(1+j)^{\alpha p}} \end{aligned}$$

Choosing $p_0 \geq \frac{1}{\alpha}$ and λ large enough, the series

$$\sum_{j \geq 0} \sum_{p \geq p_0} \left(\frac{C}{\lambda}\right)^p \frac{1}{(1+j)^{\alpha p}}$$

converges. The point (18) is then a consequence to Borel-Cantelli's lemma.

To prove 19, we have to notice that as above $|A_n(Y)|$ is dominated by terms of the form A et B the exponential inequalities yield that there exist positive constants K_1 et K_2 such that for all $\lambda > 0$ large enough,

$$P\left(\frac{1}{\sqrt{1+j}} \sup_n |A_n(Y)| > \alpha\right) \leq K_1 \exp \frac{-\lambda^2(1+j)}{K_2 M^2}.$$

Therefore, the Borel-Cantelli's lemma leads to

$$\sup_{j \geq 1} \frac{1}{\sqrt{1+j}} \sup_n |A_n(Y)| < \infty \quad p.s.$$

Or

$$2^{-j/p} \left[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right]^{1/p} \leq \sup_n |A_n(Y)|$$

Thus

$$\sup_{j \geq 1} \frac{2^{-j/p}}{p^{1/2}(1+j)^{1/2}} \left[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \right]^{1/p} \leq \frac{1}{p^{1/2}} \sup_{j \geq 1} \sup_n |A_n(Y)|.$$

and that ends the establishment of (19).

4.3 Concluding remarks

In the present paper, we have established a large deviation principle (LDP) associated of stochastic differential equation solution of (1) in the Besov-Orlicz space by using Azencott's method. This extends the LDP proved by H.Lapeyre [11] to the case of usual topology of uniform convergence. A natural extension of this work is to replace the standard brownian motion by a Fractional brownian motion W^H for every value of the Hurst parameter $H \in (0, 1)$

Competing Interests

Authors have declared that no competing interests exist.

References

- [1] Freidlin MI. The averaging principle and theorems on large deviations. Russian Math. Surveys. 1978;33(5):117-176.
- [2] Freidlin MI, Wentzell AD. Flouktouatsii v dinamitcheskikh sistemakh p oddeistviem malykh sloutchaynykh vozmouchtchenii (Fluctuations in dynamic systems under the action of small random disturbances). Naouka, Moscow; 1979.
- [3] Freidlin MI, Wentzell AD. Random perturbations of dynamical systems. Springer-Verlag; 1984.
- [4] Varadhan SRS. Large deviations and applications. SIAM, Philadelphia; 1984.
- [5] Azencott R. Large deviations and applications. Reading Notes in Math. Springer-Verlag, New York; 1980.
- [6] Strook DW. An introduction to the theory of large deviation. Spring-Verlag, Berlin. 1984;774-176..
- [7] Ben Arous G, Ledoux C. Large deviation of Freidlin-Wentzell in Hlderian norm. Reading Notes in Mathematics. 1994 ;1583 :293-299. Springer, Berlin.
- [8] Eddabhi M, Nzi M, Oukinine Y. Large deviation of the diffusion in the Besov-Orlicz space and Applications. Stochastics and Stochastics Reports. 1999;65:299-315.
- [9] Cielecki Z, Kerkycharian G, Roynette B. Some functional spaces associated with Gaussian processes. Studia Mathematica. 1993;107:171-204.
- [10] Brancovan M, Bronner F, Priouret P. Large deviations for random differential systems. Reading Notes, t. 1982 ;920. Springer-Verlag
- [11] Lapeyre H. Estimates of large deviations for systems where Gaussian noise and non-Gaussian noise appear. Ann. of the IMP, Section B. 1986;22(1):9-17.
- [12] Mellouk M. A large deviation principle for random evolution equations. Bernoulli. 2000;6(6):977-999.
- [13] Priouret P. Note on small perturbations of dynamic systems. Lectures Notes in Mathematics. t. 1982;920.
- [14] Rockafellar R. Mesurable dependance of convex sets and function on parameters. Journal of Mathematical Analysis and Applications, t. 1969;28.

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